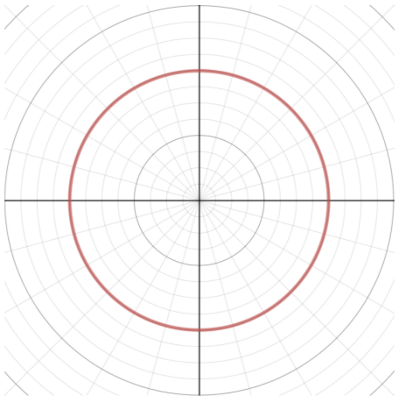
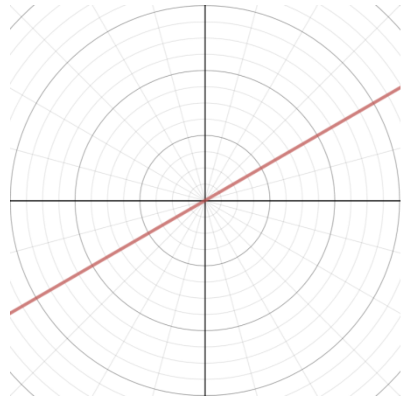
Polar Curves

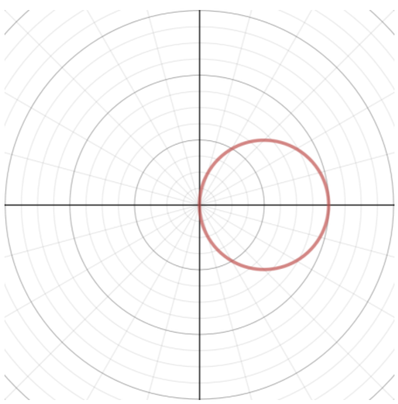
The graph of a polar equation $r = f(\theta)$ consists of all points P that have at least one polar representation (r, θ) whose coordinates satisfy the equation.



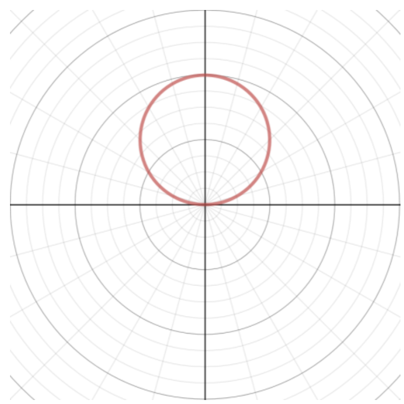
$r = 4$



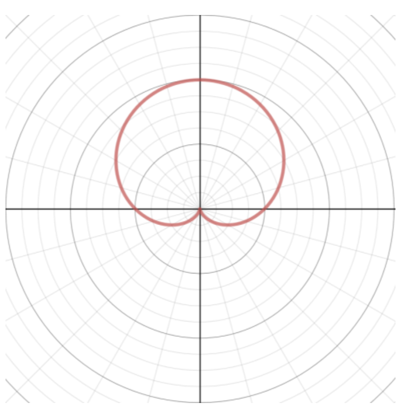
$\theta = \dfrac{\pi}{6}$



$r = 4\cos\theta$

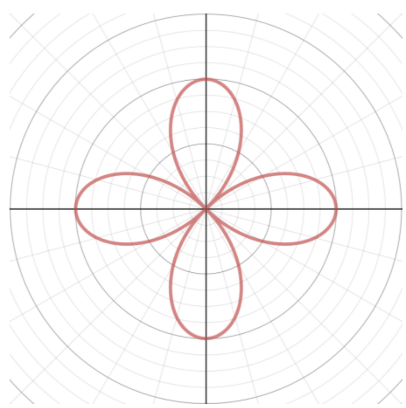


$r = 4\sin\theta$



$r = 2 + 2\sin\theta$

Cardioid



$r = 4\cos 2\theta$

Four-leaved Rose

Symmetry

When we sketch polar curves, it is sometimes helpful to take advantage of symmetry.

- The polar curve is symmetric about the **polar axis** if
  - (r,θ) and $(r,-\theta)$ are both on the curve **OR**
  - (r,θ) and $(-r,-\theta+\pi)$ are both on the curve.
- The polar curve is symmetric about the **vertical line** $\boldsymbol{\theta = \frac{\pi}{2}}$ if
  - (r,θ) and $(-r,-\theta)$ are both on the curve **OR**
  - (r,θ) and $(r,-\theta+\pi)$ are both on the curve.
- The polar curve is symmetric about the **pole** if
  - (r,θ) and $(-r,\theta)$ are both on the curve **OR**
  - (r,θ) and $(r,\theta+\pi)$ are both on the curve.

Tangents to Polar Curve

To find a tangent line to a polar curve $r = f(\theta)$, we regard θ as a parameter and write its parametric equations as

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Rightarrow \begin{cases} x = f(\theta)\cos\theta \\ y = f(\theta)\sin\theta \end{cases}$$

Then, using the method for finding slopes of parametric curves and the Product Rule, we have

Notice that if we are looking for tangent lines at the pole, then $r = 0$. Thus the above equation simplifies to

Example:

Find the slope of the tangent line of the cardioid $r = 1 + \sin\theta$ when $\theta = \dfrac{\pi}{3}$.

Write an equation of this tangent line.