Instruction: Fill in all blanks and examples. Page 4 is optional.

Arc Length

Arc Length

Copy Theorem 5, pg 653

If a curve Γ is described by the parametric equations $\begin{cases} x = h(t) \\ y = k(t) \end{cases}$, $\alpha \leq t \leq \beta$, where $h'(t)$ and $k'(t)$ are continuous on $[\alpha, \beta]$ and Γ is traversed *exactly once* as t increases from α to β, then the length of Γ is

$$L = \underline{\hspace{6cm}}.$$

Note: Γ is pronounced 'Gamma'.

Example 4 (pg 653):
Show that the circumference of the unit circle is in fact 2π using the above arc length formula. Please follow the book's method.

Extra optional exercises: Questions 41, 43, 45, 51 from page 656.

Example:

Consider the parametric curve (**cycloid**) $\Gamma : \begin{cases} x = 2(\theta - \sin\theta) \\ y = 2(1 - \cos\theta) \end{cases}$ where $0 \leq \theta \leq 2\pi$. Find the length of Γ.

This is a special case of Example 5, pg 653-654, so you can copy the book.

Surface Area

Surface Area

Copy Equation 6, pg 654.

If the curve Γ given by the parametric equations $\begin{cases} x = h(t) \\ y = k(t) \end{cases}$, $\alpha \leq t \leq \beta$, is rotated about the $x-$axis, where $h'(t)$ and $k'(t)$ are continuous and $k(t) \geq 0$, then the area of the resulting surface is given by

$$S_A = \underline{\hspace{8cm}}.$$

Example 6 (page 654):

Show using the above formula that the surface area of a sphere of radius r is in fact $4\pi r^2$. Solve the problem following the book's method.

Extra optional exrcise: Questions 61, 63, 65 from page 656.

Example(Optional):

Consider the parametric curve (**cycloid**) $\Gamma : \begin{cases} x = 2\,(\theta - \sin\theta) \\ y = 2\,(1 - \cos\theta) \end{cases}$ where $0 \leq \theta \leq 2\pi$. Find the surface area formed by rotating Γ about the $x-$axis.