(ANSWER KEY) HW Sec 7.8 Improper Integral

NAME : ________________________________

VERIFY ALL ANSWERS WITH A COMPUTING TOOL (like WolframAlpha) when possible.

1. For what values of p is $\int_1^\infty \frac{1}{x^p}\,dx$ convergent?

Answer: copy Ex 4 pg 530.

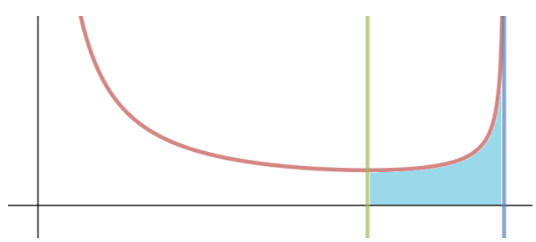
2. For what values of p is the integral $\int_0^1 \frac{1}{x^p}\,dx$ convergent?

Answer:
Do the same work as in Ex 4 pg 530 (separate the cases when p=1, p<1, and p>1) but the interval of your integral is now from 0 to 1.
When p < 1, the improper integral converges to 1/(1-p).
When p > 1 or p = 1, the improper integral diverges.

3. Evaluate $\int_0^\infty xe^{-x}\,dx$. Use integration by parts and limit laws.

Answer: 1

4. Find the area of the region enclosed by the graph of $f(x)=\dfrac{1}{x\sqrt{9-x^2}}$ and the x-axis on the interval $\left[\dfrac{3\sqrt{2}}{2},3\right]$.



Answer: Use trig substitution x = 3 sin theta.
There is an infinite discontinuity at 3, and the improper integral converges to a positive number. WolframAlpha will give you the number.

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5. Let R be the region bounded by the function $f(x)=\dfrac{1}{\sqrt{1-x^2}}$ and x-axis on the interval $[0,1]$. Evaluate the area of R.

   Answer: use trig substitution x = sin theta.
   There is an infinite discontinuity at 1, and this improper integral converges to pi/2.

6. Evaluate $\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$.

Hint: Try integration by parts either u = ln x or u = x^(-1/2).
Answer: converges to -4.

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7. Evaluate $\int_0^3 \frac{1}{x^2 - 6x + 5}\, dx$.

(Hint: Use partial fraction decomposition to write the integrand as the sum of two fractions).

Answer: Since the denominator is equal to (x-1)(x-5), you can find A and B such that the integral is equal to A/(x-1) + b/(x-5).

You can use WolframAlpha to verify your partial fraction by typing 'partial fraction of … '.

The indefinite integral is equal to ln|x-5| - ln|x-1|

There is an infinite discontinuity at x = 1, so you have to separate the improper integrals over [0,1] and over [1,3].

Both improper integrals diverge.

8. Evaluate $\int_0^{\frac{\pi}{2}} \sec^4 x\, dx$.

Answer:
Save one factor of (sec x)^2.
Apply trig substitution involving (sec x)^2.
Do u-substitution with u = tan x.
There is an infinity discontinuity at x = pi/2
The improper integral is divergent.

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9. Evaluate $\int_0^{\infty} \frac{1}{\sqrt{x}(1+x)} dx$.

Answer:
Try u-substitution with u = sqrt(x)
After the substitution, you get an integral in the form 1/(u^2 + 1).
The improper integral converges to pi.

10. If $f(t)$ is continuous for $t \geq 0$, the **Laplace Transform** of f is the function F defined by

$$F(s) = \int_0^{\infty} f(t) e^{-st}\, dt$$

and the domain of F is {all numbers s for which the improper integral converges}.
Your task: for each function below, compute its **Laplace Transform** and its domain.
(Please use WolframAlpha to check your answer. Type 'Laplace Transform of … ')

a. $f(t) = 1$ (Optional: watch https://www.khanacademy.org/math/differential-equations/laplace-transform/laplace-transform-tutorial/v/laplace-transform-1)

Answer: 1/s, domain is all positive numbers.

Domain of F =

b. $f(t) = e^t$ (Optional: watch https://www.khanacademy.org/math/differential-equations/laplace-transform/laplace-transform-tutorial/v/laplace-transform-2)

Answer: 1/(s-1), domain is all real numbers larger than 1.

Domain of F =

c. $f(t) = t$

Answer: 1/(s^2), domain is all positive numbers.

Domain of F =