Homework from Notes Sec 11.2 parts 1,2

Name : ________________________________

So far we have learned three tests for testing convergence/divergence of a SERIES:

- **Geometric Series Test: a geometric series is convergent if and only if the ratio is between -1 and 1.**
- **Divergence Test: if the sequence does not converge to 0, the series diverges.**
- **Harmonic Series Test: the harmonic series diverges.**

Please verify your answer with WolframAlpha:
https://www.wolframalpha.com/examples/mathematics/discrete-mathematics/sequences/

1. (Do at least one - pick the most challenging-looking series)
Determine whether the **geometric series** is convergent or divergent. If it is convergent, find its sum.

   a. $3-4+\frac{16}{3}-\frac{64}{9}+\quad\ldots$

   b. $2+0.5+0.125+0.03125+\quad\ldots$

   c. $\sum_{k=1}^{\infty}\frac{10^k}{(-9)^{k-1}}$

   d. $\sum_{k=1}^{\infty}\frac{(-3)^{k+1}}{4^k}$

   e. $\sum_{k=0}^{\infty}(-\pi)^k e^{-k}$

   f. $\sum_{k=2}^{\infty}\frac{2^{3k+1}}{3^{2k-1}}$

2. Consider the infinite series $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$.
   a. Determine whether the sequence $\{a_n\}$ is convergent or divergent.
   b. Determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent (if possible).

   Determine whether we can apply one of the three tests we've learned.
   - Can we apply the Geometric Series test?
   - Is the Divergence Test conclusive?
   - Can we apply the Harmonic Series Test?

3. Consider the infinite series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$.
   a. Determine whether the sequence $\{a_n\}$ is convergent or divergent.
   b. Determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent (if possible).

   Determine whether we can apply one of the three tests we've learned.
   - Can we apply the Geometric Series Test?
   - Is the Divergence Test conclusive?
   - Can we apply the Harmonic Series Test?

4. Let $a_n = \frac{n}{2n+1}$
   a. Determine whether the sequence $\{a_n\}$ is convergent or divergent.
   b. Determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent (if possible).

   Determine whether we can apply one of the three tests we've learned.
   - Can we apply the Geometric Series Test?
   - Is the Divergence Test conclusive?
   - Can we apply the Harmonic Series Test?

5. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using some of the three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum.

   a. $\frac{1}{3}+\frac{1}{6}+\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+ \quad \ldots$

   b. $\frac{1}{3}+\frac{2}{9}+\frac{1}{27}+\frac{2}{81}+\frac{1}{243}+\frac{2}{729}+ \quad \ldots$

   c. $\sum_{n=1}^{\infty} \frac{n(n+2)}{(n+3)^2}$

6. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using some of the three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum. You can use Desmos/ WolframAlpha to plot your sequence.

d. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n}$

e. $\sum_{n=1}^{\infty} \sqrt[n]{2}$

f. $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{2n+1}\right)$

7. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using some of three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum. You can use Desmos/ WolframAlpha to plot your sequence.

   g. $\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{2}{3}\right)^n}$

   h. $\sum_{n=1}^{\infty} (\cos 1)^n$

   i. $\sum_{n=1}^{\infty} \arctan n$

8. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using the geometric series test or divergence test (possibly none of them works). If it is convergent (geometric series), find its sum.

   j. $\sum_{n=1}^{\infty}\left(\frac{3}{5^n}+\frac{2}{n}\right)$

   k. $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$

   l. $\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$

   m. $\sum_{n=1}^{\infty}\left[\frac{5}{n(n+1)}-(-1)^n\frac{3}{2^n}\right]$