Selected Problems taken from Homework from Notes Sec 11.2 parts 1,2

We have learned two tests for testing convergence/divergence of a SERIES:

- **Geometric Series Test: a geometric series is convergent if and only if the ratio is between -1 and 1.**
- **Divergence Test: if the sequence does not converge to 0, the series diverges.**
- **Harmonic Series Test: the harmonic series diverges.**

1. (Pick one) Determine whether the **geometric series** is convergent or divergent. If it is convergent, find its sum.
   a. $2+0.5+0.125+0.03125+\ \ldots$ Ratio = ¼. Sum is 2(4/3) = **8/3**.
   b. $\sum_{k=0}^{\infty}(-\pi)^k e^{-k}$ Ratio = -pi/e < - 1, thus this geometric series is **divergent**.
   c. $\sum_{k=2}^{\infty}\frac{2^{3k+1}}{3^{2k-1}}$ Ratio = 8/9. Sum is 6(64/81)(9) = 6(64/9) = **128/3**.
2. Consider the infinite series $\sum_{k=1}^{\infty}\ln\left(\frac{k+1}{k}\right)$.
   a. Determine whether the sequence $\{a_n\}$ is convergent or divergent. **Convergent to 0.**
   b. Determine whether the series $\sum_{n=1}^{\infty}a_n$ is convergent or divergent (if possible).
   Determine whether we can apply one of the three tests we've learned.
      - Can we apply the Geometric Series Test? **No.**
      - Is the Divergence Test conclusive? **No.**
      - Can we apply the Harmonic Series Test? **No.**
3. Let $a_n=\frac{n}{2n+1}$
   a. Determine whether the sequence $\{a_n\}$ is convergent or divergent. **Convergent to ½.**
   b. Determine whether the series $\sum_{n=1}^{\infty}a_n$ is convergent or divergent (if possible).
      - Can we apply the Geometric Series Test? **No.**
      - Is the Divergence Test conclusive? **Yes. By the Divergence Test, the series diverges.**
      - Can we apply the Harmonic Series Test? **No.**
4. (Pick one) Attempt to determine whether the series $\sum_{n=1}^{\infty}a_n$ is convergent or divergent using some of the three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum.
   a. $\frac{1}{3}+\frac{1}{6}+\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+\ \ldots$ = **1/3 (1 + ½ + 1/3 + ¼ + 1/5 + …) is divergent by the Harmonic Series Test.**
   b. $\frac{1}{3}+\frac{2}{9}+\frac{1}{27}+\frac{2}{81}+\frac{1}{243}+\frac{2}{729}+\ \ldots$ **= 1/3 sum (1/9)^k + 2/9 sum (1/9)^k = 3/8 + 2/8.**
   c. $\sum_{n=1}^{\infty}\frac{n(n+2)}{(n+3)^2}$ **Divergent by the Divergence Test since the sequence converges to 1.**

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5. (Pick one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using some of the three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum. You can use Desmos/ WolframAlpha to plot your sequence.

   d. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n}$ **= 2/3 + 3/2 = 13/6.**

   e. $\sum_{n=1}^{\infty} \sqrt[n]{2}$ **Divergent by the Divergence Test since the sequence converges to 1.**

   f. $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{2n+1}\right)$ **Divergent by the Divergence Test since the sequence converges to a nonzero number.**

6. (Do at least one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using some of three tests we've learned (possibly none of them works). If it is convergent (geometric series), find its sum. You can use Desmos/ WolframAlpha to plot your sequence.

   a. $\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{2}{3}\right)^n}$ **Divergent by the Divergence Test since the sequence converges to 1.**

   b. $\sum_{n=1}^{\infty} (\cos 1)^n$ **= cos(1) / [1 – cos(1)]**

   c. $\sum_{n=1}^{\infty} \arctan n$ **Divergent by the Divergence Test since the sequence converges to pi/2.**

7. (Pick one) Attempt to determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent using the geometric series test or divergence test (possibly none of them works). If it is convergent (geometric series), find its sum.

   d. $\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n}\right)$ **Since sum (3/ 5^n) is equal to a number, but sum 1/n is the harmonic series (which is divergent), the entire sum is divergent.**

   e. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ **Divergent by the Divergence Test since the sequence doesn't converge to 0 (in fact, the sequence diverges).**

   f. $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$ **Divergent by the Divergence Test since the sequence converges to 1.**

   g. $\sum_{n=1}^{\infty} \left[\frac{5}{n(n+1)} - (-1)^n \frac{3}{2^n}\right]$ **None of the three tests can be applied.**