1. Evaluate $\int_0^8 \frac{1}{\sqrt[3]{x}}\,dx$.

[Solution]

$$\begin{aligned}\int_0^8 \frac{1}{\sqrt[3]{x}}\,dx &= \lim_{t\to 0^+}\int_t^8 \frac{1}{\sqrt[3]{x}}\,dx \\ &= \lim_{t\to 0^+}\left(\frac{3}{2}x^{\frac{2}{3}}\right)_t^8 \\ &= \frac{3}{2}\lim_{t\to 0^+}\left(8^{\frac{2}{3}} - t^{\frac{2}{3}}\right) \\ &= 6\end{aligned}$$

2. Evaluate $\int_0^{\frac{\pi}{2}} \sec x\,dx$.

[Solution]

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sec x\,dx &= \lim_{t\to\left(\frac{\pi}{2}\right)^-}\int_0^t \sec x\,dx \\ &= \lim_{t\to\left(\frac{\pi}{2}\right)^-}\left(\ln\left|\sec x + \tan x\right|\right)_0^t \\ &= \lim_{t\to\left(\frac{\pi}{2}\right)^-}\ln\left|\sec t + \tan t\right| \\ &= \infty\end{aligned}$$

The improper integral $\int_0^{\frac{\pi}{2}} \sec x\,dx$ diverges.

3. Evaluate $\int_0^5 \frac{x}{x-2}\,dx$.

[Solution]

$\int_0^5 \frac{x}{x-2}\,dx = \lim_{t\to 2^-}\int_0^t \frac{x}{x-2}\,dx + \lim_{s\to 2^+}\int_s^5 \frac{x}{x-2}\,dx$

Let $u = x-2$, then $du = dx$.

When $x=0$, $u=-2$ and when $x=t$, $u=t-2$.

$$\begin{aligned}
\text{Thus } \lim_{t\to 2^-}\int_0^t \frac{x}{x-2}\,dx &= \lim_{t\to 2^-}\int_{-2}^{t-2} \frac{u+2}{u}\,du \\
&= \lim_{t\to 2^-}\int_{-2}^{t-2}\left(1+\frac{2}{u}\right)du \\
&= \lim_{t\to 2^-}\left(u+2\ln|u|\right)_{-2}^{t-2} \\
&= \lim_{t\to 2^-}\left[(t-2)-(-2)+2\ln|t-2|-2\ln 2\right] \\
&= -\infty
\end{aligned}$$

The improper integral $\int_0^5 \frac{x}{x-2}\,dx$ diverges.

4. Use the Comparison Theorem to determine whether the integral $\int_1^\infty \frac{x}{x^3+1}\,dx$ is convergent or divergent.

[Solution]

Note that $x^3+1 > x^3$ for all x.

Thus $\frac{1}{x^3+1} < \frac{1}{x^3}$ for all x.

That is $\frac{x}{x^3+1} < \frac{x}{x^3}$ for all x.

$$\begin{aligned}
\text{Therefore, } \int_1^\infty \frac{x}{x^3+1}\,dx < \int_1^\infty \frac{x}{x^3}\,dx &= \int_1^\infty \frac{1}{x^2}\,dx \\
&= \lim_{t\to\infty}\int_1^t \frac{1}{x^2}\,dx \\
&= \lim_{t\to\infty}\left(-\frac{1}{x}\right)_1^t \\
&= -\lim_{t\to\infty}\left(\frac{1}{t}-1\right) \\
&= 1
\end{aligned}$$

As a result, the improper integral $\int_1^\infty \frac{x}{x^3+1}\,dx$ converges.