HW 11.10

3. The Maclaurin series for $\arctan x$ is $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for $-1 < x < 1$.

   a. Determine the Maclaurin series for $x^3 \arctan x$. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+4}}{2n+1}$ for $|x|<1$.

   b. Determine the Maclaurin series for $\int x^3 \arctan x \, dx$. (Hint: use term-by-term integration, Sec 11.9). $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+5}}{(2n+1)(2n+5)} + C$ with radius of convergence 1

   c. Determine a series that represents $\int_0^{0.1} x^3 \arctan x \, dx$. (Hint: follow Example 11b pg 769).

   d. If the first two non-zero terms of the series are used to estimate the value of the definite integral from the previous part, provide a bound on the error of this estimate. (Hint: follow Example 11b pg 769).

c) Since $0.1 < 1$, we have

$$\int_0^{0.1} x^3 \arctan x \, dx = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+5}}{(2n+1)(2n+5)} \Bigg|_{x=0}^{x=0.1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(0.1)^{2n+5}}{(2n+1)(2n+5)}$$

note: ← alternating series because $(-1)^n$

$$= \frac{(0.1)^5}{1 \times 5} - \frac{(0.1)^7}{3 \times 7} + \frac{(0.1)^9}{5 \times 9} - \frac{(0.1)^9}{7 \times 11} + \cdots$$

first two nonzero terms

d) By the Alternating Series Estimation Thm, the error $\left| \left( \int_0^{0.1} x^3 \arctan x \, dx \right) - \sum_{n=0}^{1} \frac{(-1)^n (0.1)^{2n+5}}{(2n+1)(2n+5)} \right|$ is bounded by $\frac{(0.1)^9}{5 \times 9}$

**Homework due Week 11, day 1 Tue** (Please use your own paper)

Name : ______________________________

1. Find the Maclaurin series for $f$ using the definition. Verify your answer with Table 1 and Wolfram|Alpha.
   a. $f(x) = \dfrac{1}{1-x}$
   b. $f(x) = \ln(1+x)$
   c. $f(x) = 2^x$

2. Find the Taylor series for $f(x)$ centered at $x = a$.
   a. $f(x) = x^4 - 3x^2 + 1$ and $a = 1$
   b. $f(x) = \sqrt{x}$ and $a = 16$. Hint: See Example 8 on page 766. See also an almost identical problem: overleaf.com/read/krtzsqgykktb

3. The Maclaurin series for $\arctan x$ is $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1}$ for $-1 < x < 1$.
   a. Determine the Maclaurin series for $x^3 \arctan x$.
   b. Determine the Maclaurin series for $\int x^3 \arctan x \, dx$. (Hint: use term-by-term integration, Sec 11.9).
   c. Determine a series that represents $\int_0^{0.1} x^3 \arctan x \, dx$. (Hint: follow Example 11b pg 769).
   d. If the first two non-zero terms of the series are used to estimate the value of the definite integral from the previous part, provide a bound on the error of this estimate. (Hint: follow Example 11b pg 769).

**Homework**

4. Do Sec 11.10 Example 12 on pg 769 using *series*. Close the book while you work out the answer, and verify with the book afterwards.

5. (a) Use *series* to evaluate the limit. Use Table 1. (b) Verify your answer using either L'hopital rule or the computer. (Hint: follow Example 12 pg 769).

$$\lim_{x \to 0} \frac{x - \ln(1+x)}{x^2}$$

6. (a) Use *series* to evaluate the limit. Look up the relevant Maclaurin series from Table 1. (b) Verify your answer using either L'hopital rule or a computer. (Hint: follow Example 12 pg 769).

   a. $\lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x}$

   b. $\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$