MATH 1152Q Exam 1 Summary

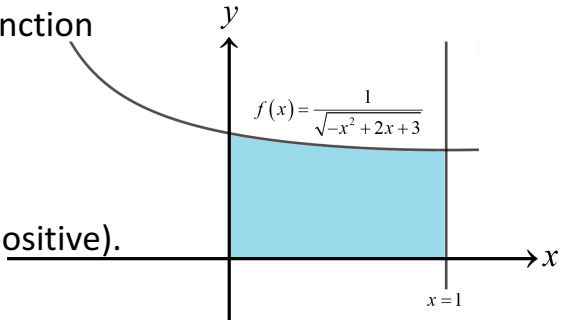
[Sec 7.2-7.3] Techniques of Integration

【7.2】Trigonometric Integrals (trig identities will be provided)

1. Evaluate $\int \frac{\cos^2(\arctan x)}{1+x^2}\,dx$. (Hint: I prefer drawing a triangle for this one)
2. Evaluate $\int \sin^3 x \cos^4 x\,dx$.
3. Evaluate $\int \sin^4 x \cos^3 x\,dx$.
4. Evaluate $\int \tan^3 x \cdot \sec^2 x\,dx$.
5. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{1-\cos 2x}\,dx$.
6. Evaluate $\int \frac{1-\tan^2 x}{1+\tan^2 x}\,dx$.

【7.3】Trigonometric Substitution (trig identities and table of substitution will be provided)

1. Let R be the region bounded by the function $f(x)=\frac{1}{\sqrt{-x^2+2x+3}}$ and x-axis on the interval $[0,1]$. Compute the area of R.
   (Sanity check: your answer should be positive).



2. Evaluate $\int_1^e \frac{1}{y\sqrt{1+(\ln y)^2}}\,dy$. Sanity check: is your answer positive?
3. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1+\sin^2 x}}\,dx$. Sanity check: is your answer positive?
4. Evaluate $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$.
5. Evaluate $\int_{\sqrt{2}}^2 \frac{1}{x^3\sqrt{x^2-1}}\,dx$.

MATH 1152Q Exam 1 summary Answer

[Sec 7.2-7.3] Techniques of Integration

【7.2】Trigonometric Integrals

(1) $\frac{1}{2}\arctan x+\frac{x}{2\left(1+x^2\right)}+C$

(2) $-\frac{1}{5}\cos^5 x+\frac{1}{7}\cos^7 x+C$

(3) $\frac{1}{5}\sin^5 x-\frac{1}{7}\sin^7 x+C$

(4) $\frac{1}{4}\tan^4 x+C$

(5) $\sqrt{2}$

(6) $\frac{1}{2}\sin 2x+C$

【7.3】Trigonometric Substitution

(1) $\frac{\pi}{6}$

(2) $\ln\left(1+\sqrt{2}\right)$

(3) $\ln\left(\sqrt{2}+1\right)$

(4) $\operatorname{arcsec}|x-1|+C$

(5) $\frac{\pi}{24}+\frac{\sqrt{3}}{8}-\frac{1}{4}$