MATH 1152Q Exam 1 Summary

[11.1-11.2, 11.4-11.6] Infinite Sequences and Series

【11.1】Sequences

1. Determine whether the ***sequence*** $\{a_n\}$ is convergent to a number.

   a. $a_n = \dfrac{3^{n+2}}{5^n}$

   b. $a_n = \sqrt[2n]{e^{n+2}}$

   c. $a_n = \dfrac{(\ln n)^2}{n}$

   d. $a_n = \dfrac{\cos^2 n}{2^n}$

【11.2】Series

1. Determine if the series is convergent of divergent. If it is convergent, find its sum.

   a. $\sum_{n=1}^{\infty} \left(\dfrac{e}{\pi}\right)^n$. You don't need to simplify.

   b. $\sum_{n=1}^{\infty} \ln\left(\dfrac{n+1}{n}\right)$. Hint: write as a telescoping sum.

   c. $\sum_{n=1}^{\infty} \left[\left(\dfrac{e}{\pi}\right)^n + \ln\left(\dfrac{n+1}{n}\right)\right]$

2. Express 0.10181818181818 … as a ratio of two integers. You don't need to simplify.

【11.2】The Divergence Test

1. Let $a_n = \dfrac{n^2}{2n+1} - \dfrac{n^2}{2n-1}$.

   a. Determine whether the sequence $\{a_n\}$ is convergent or divergent. Hint: simplify.

   b. Determine whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or divergent.

【11.6】The Ratio Test

1. Use the **Ratio Test** to determine whether the series $\sum_{n=1}^{\infty} \dfrac{n3^n}{(2n+1)!}$ is convergent.

【11.4】The 2 Comparison Tests

1. Use the **Limit Comparison Test (or the comparison test)** to determine whether the series $\sum_{n=1}^{\infty} \frac{\cos^2 n + 5n}{8^n}$ is convergent or divergent.
2. Use the **Limit Comparison Test (or the comparison test)** to determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$ is convergent or divergent.
3. Consider the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$.

   a. Use the **Divergence Test** to determine whether the series $\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$ is convergent.

   b. Use the **Limit Comparison Test** to determine whether the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ is convergent or divergent.
4. Let $\{p_n\}_{n=1}^{\infty}$ be a sequence of prime numbers. You are told that $\lim_{n\to\infty} \frac{p_n}{n\ln n} = 1$, the famous Prime Number Theorem. Use the **Limit Comparison Test** to determine whether the series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ is convergent or divergent. You can assume that $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ is divergent (a fact that will be discussed later this semester).

【11.5】Alternating Series

1. Use the **Alternating Series Test** to determine the convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$.
2. Determine whether the series $\sum_{n=1}^{\infty} \cos(n\pi)\tan\left(\frac{\pi}{n}\right)$ is convergent or divergent.

【11.6】More alternating series

Determine whether the series is convergent or divergent.

a. $\sum_{n=1}^{\infty} (-1)^{n+1} \arctan n$

b. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

c. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh(n)}$. Think of cosh as just a `mystery function'. You may use the fact that cosh(n) is an increasing sequence for *n=1,2,3, ….* It is not necessary to visualize this, but you can see graph: https://www.desmos.com/calculator/e8ypqenzq7

MATH 1152Q Exam 1 Summary Answer

[11.1,2,4,5,6] Infinite Sequences and Series

【11.1】Sequences

(1) (a) The sequence converges to 0. (b) The sequence converges to $\sqrt{e}$.

(c) The sequence converges to 0. (d) The sequence converges to 0.

【11.2】Series

(1) (a) The series converges to $\frac{e}{\pi - e}$. (b) The series diverges. (c) The series diverges. (2) $\frac{28}{275}$

【11.2】The Divergence Test

(1) (a) The sequence converges to $-\frac{1}{2}$. (b) The series diverges.

【11.6】The Ratio Test

(1) The series converges.

【11.4】The 2 Comparison Tests

(1) The series converges. (2) The series converges. (3) (a) The series diverges. (b) The series diverges. (4) The series diverges.

【11.5】Alternating Series

(1) The series converges. (2) The series converges.

【11.6】More alternating series

(a) The series is divergent. (b) The series is convergent.

(c) The series is convergent.