Name: ______________________________ **Math 1152Q: Fall '17**
Week 4 Sample Quiz

Summary: Sec 11.1 (limit computation only), Sec 11.2 (knowing whether geometric series, harmonic series converge), Sec 11.4 (comparison tests for series with positive terms only), Sec 11.6 (ratio test for series with positive terms only, but it's OK if you use the root test instead). **All terms of the series will be positive.**

STRATEGY TIPS:

✓ The ratio test usually works when the term contains factorial like $(n+3)!$ or exponents like 7^n, $\frac{1}{7^n}$.

✗ The ratio test will *not* work with series with ONLY p-series-like terms, for example, $\sum \frac{n^2+4}{\sqrt{n^5-1}}$. Convince yourself.

✓✗ Only use one of the comparison tests are when the series looks like the geometric series $\sum r^n$, or the p-series $\sum \frac{1}{n^p}$.

You can check all the 'does [blank] converge' questions below with WolframAlpha.

1. **(Statement of theorem - guaranteed to show up in the quiz)**
   (a) Write the statement of the *divergence test* as stated in Stewart Sec 11.2 (either box no. 6 or 7 is OK).
   (b) Write the statement of the *comparison test* as stated in Stewart Sec 11.4.
   (c) Write the statement of the *limit comparison test* as stated in Stewart Sec 11.4.
   (d) Write the statement of the *ratio test* as stated in Stewart Sec 11.6 **(assume the terms of the series are positive).**
   (e) (You do not need to memorize the statement of the *root test* but you may use it on a test if you want.)
   (f) (Sec 11.4 WebAssign no. 1) Let $\sum a_n$ and $\sum b_n$ be series with positive terms.
      (a) Suppose $\sum a_n \leq \sum b_n$ for all n. We know that b_n is convergent. What can you say about $\sum a_n$?
      (b) Suppose $\sum a_n \geq \sum b_n$ for all n. We know that b_n is convergent. What can you say about $\sum a_n$?
      (c) Suppose $\sum a_n \leq \sum b_n$ for all n. We know that b_n is divergent. What can you say about $\sum a_n$?
      (d) Suppose $\sum a_n \geq \sum b_n$ for all n. We know that b_n is divergent. What can you say about $\sum a_n$?

2. **(Book examples - at least one of these will show up on the quiz)**
   Pages 728-730: Sec 11.4 Examples 1,2,3,4
   Pages 740-741 Sec 11.6 Examples 3, 5 (assume all terms are positive).

3. Show whether each series $\sum a_n$ below converges or diverges using the Comparison Test or Limit Comparison Test. For 'excellent' mark you should give
   - The series $\sum b_n$ with which you compare and a short statement on why it converges or diverges

- An inequality or limit computation
  - If using the Comparison Test, give an inequality of the form $a_n \le b_n$ or $a_n \le b_n$
  - If using the Limit Comparison Test, compute $\lim_{n\to\infty} a_n/b_n$
- A conclusion statement.

(a) $\sum_{n=2}^{\infty} \dfrac{n^3}{n^4-1}$

(b) $\sum_{n=1}^{\infty} \dfrac{6^n}{5^n-1}$

(c) $\sum_{n=1}^{\infty} \dfrac{(2n-1)(n^2-1)}{(n+1)(n^2+4)^2}$

4. (WebAssign Sec 11.4)

(a) (no. 2) Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{6n^3+1}$ converges or diverges. (Hint: compare with a p-series)

(b) (no. 3) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{6}{\sqrt{n^2+5}}$ converges or diverges. (Hint: compare with a p-series)

(c) (no. 4) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{n+6^n}{n+2^n}$ converges or diverges.
Hints: ✓LCT attempt 1: You try LCT with $\sum(\frac{6}{2})^n$ and it works.
✓LCT attempt 2: LCT with $\sum \frac{1}{n}$ also works. But this may not be the first thing that comes to your mind.
✓Divergence test: the terms are increasing, so this test works.
✓Comparison test: find a big enough constant A so that $a_n > A(\frac{6}{2})^n$ - see WebAssign solution.
✓Ratio test: you see powers, so you try the ratio test. The ratio $\frac{a_{n+1}}{a_n}$ goes to 6/2.

(d) (no. 4) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{2n+3^n}{2n+7^n}$ converges or diverges.
Hints: ✓LCT attempt 1: You try LCT with $\sum(\frac{3}{7})^n$ and it works.
✓LCT attempt 2: LCT with $\sum \frac{1}{n^2}$ also works, but this may not be the first thing that comes to your mind.
✗Divergence test: inconclusive.
✓Comparison test: find a big enough constant A so that $a_n < A(\frac{3}{7})^n$ - see WebAssign solution.
✓Ratio test: you see powers, so you try the ratio test. The ratio $\frac{a_{n+1}}{a_n}$ goes to 3/7.

(e) (no. 5) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{5n^2-1}{6n^4+7}$ converges or diverges. (Hint: compare with a p-series and use one of the comparison tests. Would the ratio test be conclusive? See strategy at the top of this file.)

(f) (no. 6) Determine whether the series $\sum_{n=6}^{\infty} \dfrac{n-5}{n7^n}$ converges or diverges. (Hint: compare with a geometric series. You can also try the ratio test because you see powers $(\frac{1}{7})^n$)

(g) (no. 6) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{5^n}{n7^n}$ converges or diverges. (Hint: compare with a geometric series. You can also try the ratio test because you see powers $(\frac{5}{7})^n$)

(h) (no. 6) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{5^{2n}}{n7^n}$ converges or diverges.
Hints:

✗LCT with geometric series $\sum(\frac{25}{7})^n$ is inconclusive.
✓LCT with comparing with the harmonic series $\sum \frac{1}{n}$ works.
✓You try the ratio test because you see powers $(\frac{25}{7})^n$.

(i) (no. 7) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{n+8}{n\sqrt{n}}$ converges or diverges.

(Hints:
✓LCT: you compare with a p-series because it looks like one.
✗Ratio test: you try ratio test and it's inconclusive. The top of this file tells you that the ratio test never works for any series that looks ONLY like a p-series.)

5. (Sec 11.1 limit computation)

(a) (See l'Hopital handout pages 4-5 or WebAssign no. 5)
Compute

$$\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^n \quad \text{and} \quad \lim_{n\to\infty}\left(1+\frac{5}{4n}\right)^n$$

if they exist. (Hint: Notice the indeterminate form of type "1^∞".)

(b) (WebAssign Sec 11.1 no. 6) Determine whether the sequence

$$\left\{\frac{5n!}{2^n}\right\}_{n=1}^{\infty} \quad \text{converges or diverges.}$$

6. (Divergence Test Sec 11.2)

(a) True or false? If a_n does not converge to 0, then the series of $\sum_{n=1}^{\infty} a_n$ diverges.

(b) True or false? If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

(c) Let $a_n = \frac{4n}{7n+1}$. Determine whether $\{a_n\}$ is convergent. Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.

(d) Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2-1}{100+5n^2}$ is convergent or divergent.

(e) Let $\{a_n\}$ be a sequence which converges to 0. With this information, can you determine whether $\sum_{n=1}^{\infty} a_n$ is convergent? Explain briefly.

7. (WebAssign Sec 11.6: mainly ratio test, but also divergence test and the comparison tests)

Determine whether each of the following series $\sum a_n$ converges or diverges. For 'excellent' mark you should give

- The name of the test you use.
- The inequality (if you use the non-limit comparison test) or limit computation.
- A conclusion statement.

(a) $\sum_{n=1} \dfrac{5n!}{2^n}$ (hint: see factorial, think ratio test)

(b) (no. 2) $\sum_{n=1} \dfrac{n}{5^n}$

hints: ✗LCT with geom. series $\sum(\frac{1}{5})^n$ is inconclusive.
✓LCT comparing with $\sum \frac{1}{n^2}$ works.
✓You see power $(\frac{1}{5})^n$, so you try ratio test.

(c) (variation of no. 2) $\sum_{n=1} ne^{-5n}$ (see previous hints)

(d) (no. 3) $\sum_{n=1} \left(\dfrac{1}{4n+1}\right)^n$

Hints:
✓LCT: compare with p-series like $\sum \frac{1}{n^2}$.
✓LCT: compare with geometric series like $\sum \frac{1}{4^n}$.
✓Can use both ratio test and root test because you see powers something^n, but the computation for the ratio test is long.

(e) (no. 4) $\sum_{n=1} n \left(\dfrac{5}{7}\right)^n$ (hint: see $(\frac{5}{7})^n$, but the two comparison tests with geometric series $\sum(\frac{5}{7})^n$ are inconclusive, so you try ratio test because you see power $(\frac{5}{7})^n$. If you really want to use a comparison test, you can compare this with the p-series $\sum \frac{1}{n^2}$.)

(f) (no. 5) $\sum_{n=1} \dfrac{|\sin(5n)|}{5^n}$

(hint: ✓see sin and $(\frac{1}{5})^n$, so think comparison test with the geometric series $\sum(\frac{1}{5})^n$.
✗The LCT with $b_n = (\frac{1}{5})^n$ fails.
✓The LCT with $b_n = \frac{1}{n^2}$ works.
✗Ratio test fails.)

(g) (variation of no. 5) $\sum_{n=1} \dfrac{|\sin(5n)|}{n^5}$

(hint: ✓see sin and $(\frac{1}{n^5})$, so think the (non-limit) comparison test with the p-series $\sum(\frac{1}{n^5})$.
✗The LCT with $b_n = (\frac{1}{n^5})$ fails.
✓The LCT with $\frac{1}{n^3}$ works.
✗Ratio test fails.)

(h) (no. 6) $\sum_{n=1} \dfrac{2^n}{n^3}$.

Hints: ✓Divergence test: numerator grows faster than the denominator, so use divergence test - see WebAssign's solution.
✗LCT: you see 2^n, but find that the comparison tests with the geometric series $\sum 2^n$ are inconclusive.
✓LCT: you try LCT with $\sum \frac{1}{n}$ and find that it works.
✓Ratio test: you can try ratio test because you see power 2^n.
✓Root test (not required to memorize): you can try root test because you see power 2^n.

(i) (no. 7) $\sum_{n=1} \dfrac{n!}{100^n}$

Hints: ✓Ratio test: you see *factorial* and exponent 100^n, so think ratio test (WebAssign's solution).
✓Divergence test: you remember than factorial grows faster than exponential - see sol of WebAssign Sec 11.1 no. 6.
✗LCT: You try comparing it with $\sum n!$ but the result is inconclusive.

(j) (no. 8) $\sum_{n=1} \dfrac{n}{\sqrt{n^3+4}}$

(Hints: ✓LCT or comparison: looks like a p-series, so use either (see WebAssign sol).
✗Ratio test: you attempt the ratio test, and you get an inconclusive result. But I've told you above that the ratio test will not work for any series that look like a p-series.)

8. (Bonus, for an extra token) Write 10 or more distinct (correctly spelled) first names (not your own name) of students in this class and be able to recognize ten students with these names. Each incorrect name would negate a correct name.