Extra HW Week 4 Quiz (required to pass week 4 quiz - optional if you already passed)

Compute (do your work on scratch paper)

$$\lim_{x\to\infty} \left(\frac{x+1}{x}\right)^x = \underline{\hspace{1cm}} \qquad \text{and} \qquad \lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x = \underline{\hspace{1cm}}.$$

You can use them below without explanation.

1. (Ratio Test)

   (a) Complete the statement of the *ratio* test:

   Suppose a_n is a positive number for all $n \geq 1$. Then ...

   (b) Show your work for attempting to apply the ratio test to the series

   $$\sum_{n=1}^{\infty} \frac{n^n}{n!\, 3^n}.$$

   If the ratio test is conclusive, determine whether the series is convergent. Otherwise, state that the ratio test is inconclusive.

   (c) Show your work for attempting to apply the ratio test to the series

   $$\sum_{n=1}^{\infty} \frac{n^n\, 2^n}{n!\, 5^n}.$$

   If the ratio test is conclusive, determine whether the series is convergent. Otherwise, state that the ratio test is inconclusive.

(There is one more part of question 1 on the next page)

(d) Show your work for attempting to apply the ratio test to the series

$$\sum \frac{n^4 \ 25^n}{n!}$$

to determine whether it converges or diverges. If the ratio test is conclusive, determine whether the series is convergent or divergent. Otherwise, state that the ratio test is inconclusive.

2. (Choose your own tests)

   (a) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2n + 3^n}{2n + 7^n}$$

   converges. Clearly state each test you use and write its statement (only the relevant parts).

   (b) Determine whether the series

$$\sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}.$$

   converges. Clearly state each test you use and write its statement (only the relevant parts).

(There is one more part of question 2 on the next page)

(c) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n + 5^n}{n^9 + 2^n}$$

converges or diverges. Clearly state each test you use and write its statement (only the relevant parts).

3. (Limit Comparison Test) For each series below, use the limit comparison test (LCT) to determine whether the following series converges or diverges (LCT also includes Sec 11.4 Exercise 40 and 41 pages 732).

   If you prefer, you can use the comparison test, but you must use one of the *comparison* tests.

   (a)
   $$\sum_{n=1}^{\infty} \frac{\sqrt{n^5 + n^2 + 4}}{2n^2 - 7n}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

   (There are two more parts of question 3 on the next page)

(c)

$$\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}\, 2^n}$$

(d)

$$\sum_{n=2}^{\infty} \frac{1}{\ln n}$$