Name: ____ **Math 1152Q: Fall '17**
Week 3 Sample Quiz

All technology (phone, calculator, anything that has a computer) should be stored in your bag.

Summary: Sec 11.1 (focusing on l'Hopital rule, monotonic sequence theorem, squeeze theorem) and Sec 11.2 (focusing on decimal expansion, telescoping sum and divergence test).

1. (Vocabulary)
   (a) (Sec 11.1 Def 2 pg 696) Given a sequence $\{a_n\}$ What does $\lim_{n\to\infty} a_n = L$ mean? Use Def 2, with ϵ and N.
   (b) (Sec 11.1 Def 5 page 697) Let $\{b_n\}$ be a sequence. What does it mean to write $\lim_{n\to\infty} b_n = \infty$? Use Def 5, with M and N. Warning: do not include variations of the words "converge", "diverge", "approach", "increase", "continuously", or "infinity" in your answer.
   (c) (Sec 11.2 Def 2 pg 708) Let $\{c_n\}$ be a sequence. What is a partial sum of $\{c_n\}$?
   (d) (Sec 11.2 Def 2 pg 708) Let $\{c_n\}_{n=1}^{\infty}$ be a sequence. We say that the infinite series $\sum_{n=1}^{\infty} c_n$ is *convergent* if

   ____. If the above blank is not true, then we say that $\sum_{n=1}^{\infty} c_n$ is *not convergent* or *divergent.*

2. (Sec 11.1 monotone convergence theorem) Recall that lower and upper bounds are not unique!
   (a) True or false? The sequence
   $$\left\{\frac{3n-6}{6n+2}\right\}_{n=1}^{\infty}$$
   is bounded. Give *an* upper bound and *a* lower bound (if true) or justify (if false). (Hint: see WebAssign no. 7).
   (b) True or false? The sequence
   $$\left\{5ne^{-6n}\right\}_{n=1}^{\infty}$$
   is bounded. Give *an* upper bound and *a* lower bound (if true) or justify (if false). (Hint: see WebAssign no. 8).
   (c) True or false? Every bounded sequence is convergent. Justify (if true) or give a counterexample (if false).
   (d) True or false? There exists a monotonically increasing sequence that converges to 10. Provide an example (if true) or justify (if false).
   (e) True or false? There exists a monotonically increasing bounded sequence that does not converge (that is, diverges). Provide an example (if true) or justify (if false).
   (f) T or F? There is a non-monotonic sequence that converges to 4. Provide an example (if true) or justify (if false).

3. (Sec 11.1 proving convergence of a sequence) SEE EXAMPLE `https://egunawan.github.io/fall17/notes/notes11_1choosingN.pdf`
   (a)
   - Give *a* positive number N such that, $\dfrac{1}{n^2-5} < \frac{1}{7000}$ for all $n > N$.
   - Then generalize this. Suppose I give you a positive number ϵ. Choose *a* positive number N such that, if $n > N$, then $\dfrac{1}{n^2-5} < \epsilon$.

   (b) The sequence $a_n = (2n+4)/(5n-8)$ converges to $2/5$. Choose N so that $|2/5 - a_n| < \frac{1}{10}$ whenever $n > N$. Generalize this: for any (small) number $\epsilon > 0$, choose N so that if $n > N$, then $|2/5 - a_n| < \epsilon$.
   (c) The sequence $a_n = (n^2+1)/(7n^2+5)$ converges to $1/7$. For any (small) number $\epsilon > 0$, find N so that $|1/7 - a_n| < \epsilon$ as long as $n > N$.

4. (Sec 11.1 computation)
   (a) Show that
   $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e.$$
   (Hint: Note the indeterminate form of type 1^{∞}. See l'Hopital handout pages 4-5 and WebAssign Sec 11.1 no. 5.)
   (b) Compute
   $$\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^n \quad\text{and}\quad \lim_{n\to\infty}\left(1+\frac{5}{4n}\right)^n$$
   if they exist. (Hint: plot these sequences using Desmos to guess the limit or WolframAlpha to calculate the limit. Notice the indeterminate form of type "1^{∞}". See l'Hopital handout pages 4-5 or WebAssign no. 5 for similar examples.)

(c) Determine whether the sequence

$$\left\{\frac{5n!}{2^n}\right\}_{n=1}^{\infty} \text{ converges or diverges. (Hint: WebAssign Sec 11.1 no. 6)}$$

5. (Questions similar to WebAssign homework Sec 11.2)

(a) Compute the limit of partial sums. For example: Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$ whose *partial sum* is known to be $s_n = 4 - 7(3/10)^n$.

(b) (The divergence test) Examples:

1. • True or false? If a_n does not converge to 0, then the series of $\sum_{n=1}^{\infty} a_n$ diverges.
   • True or false? If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
2. Let $a_n = \frac{4n}{7n+1}$. Determine whether $\{a_n\}$ is convergent. Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.
3. Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2-1}{100+5n^2}$ is convergent or divergent.
4. Let $\{a_n\}$ be a sequence which converges to 0. With this information, can you determine whether $\sum_{n=1}^{\infty} a_n$ is convergent? Explain briefly.

(c) Telescoping sum and partial fraction decomposition method (where each term's denominator is a quadratic polynomial). E.g.: Determine whether each series is convergent or divergent by expressing s_n as a telescoping sum.

1. $$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}. \quad \text{Solution: see Sec 11.2 Example 8.}$$
2. $$\sum_{n=2}^{\infty} \frac{\text{some number}}{n^2-1}. \text{ (Hint: see Sec 11.2 Example 8, or WebAssign no. 5 solution video.)}$$
3. See also example from handout Sec 11.2 part 2.

(d) Find ratio values x such that the corresponding geometric series converge. Then compute the sum of the series (assuming x satisfies the condition). For example:

1. $\sum_{n=1}^{\infty} (-4)^n x^n$.
2. $\sum_{n=1}^{\infty} (-4)^{n-1} x^n$.
3. $\sum_{n=1}^{\infty} 5(-4)^{n-1} x^n$.
4. $\sum_{n=0}^{\infty} 4^{n-1} x^n$.

Solution: For each, the interval is $\left(-\frac{1}{4}, \frac{1}{4}\right)$. Don't forget to compute the sum of the series! Check your answers with WolframAlpha.

(e) Due to the geometric series formula, we know that every repeating decimal expansion can be written as a fraction.
Check that $0.\overline{3} = 0.3333\ldots$ is $1/3$ using geometric series.
Check that $0.\overline{6} = 0.6666\ldots$ is $2/3$ using geometric series.
Use the geometric series formula to write the fraction for $0.\overline{9} = 0.999\ldots$.
Show the work involved in setting up the geometric series. Make clear what your ratio is and what the first term of your series is.

6. Apply squeeze theorem to answer the following questions.

(a) Determine whether

$$\left\{\frac{1}{5\ n!\ 2^n}\right\} \text{ converges or diverges.}$$

(Hint: squeeze this sequence between 0 and a geometric sequence. See also a related sequence on WebAssign Sec 11.1 no. 6).

(b) Consider the sequence $\left\{\frac{\cos n}{\sqrt{n}}\right\}$ for $n \geq 1$. Use the squeeze theorem to show that this converges and find its limit.

Hint: Find two sequences of the type $\frac{\text{(a constant)}}{\sqrt{n}}$ that bound $\frac{\cos n}{\sqrt{n}}$. You may assume that $\lim_{n\to\infty} \frac{\text{(a constant)}}{\sqrt{n}} = 0$.