**Name:** ____________________________________________ **Math 1152Q: Fall '17**

**Week 2 Sample Quiz**

*All technology (phone, calculator, anything that has a computer) should be stored in your bag.*

Summary: From Sec 11.1, all of WebAssign Sec 11.1 (except monotonic sequence theorem) and questions similar to those listed below. From Sec 11.2, only the geometric series problems similar to the ones listed below.

1. (Vocabulary)
   (a) (Sec 2.5) Let $f$ be a function and let $a$ be a real number. According to Stewart's definition, $f$ is called *continuous* at $a$ if: $f(a)$ is ______________ , ______________ exists, and $f(a) =$ ______________.
   Warning: a function does not need a derivative at $a$ to be continuous at $a$.
   (b) (Sec 11.1) We've discussed that
   $$\lim_{n\to\infty} \frac{4n^3 - 8n - 6}{n^2 - 6n^2 + 17n^3} = \frac{4}{17}. \tag{1}$$
   What does (1) mean (according to the definition of limit of a sequence)? Warning: do not include variations of the words "converge", "diverge", "approach", or "infinity" in your answer.
   Another warning: in general $\lim_{n\to\infty} d_n = L$ does *NOT* imply that "$d_n$ will never reach $L$".
   (c) (Sec 11.1) Let $\{b_n\}$ be a sequence. What does it mean to write $\lim_{n\to\infty} b_n = \infty$? Warning: do not include variations of the words "converge", "diverge", "approach", "increase", "continuously", or "infinity" in your answer.

2. (Sec 11.1 finding closed-form formula)
   (a) Find a closed-form formula for the $n$th term in the sequence $\left\{-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \ldots\right\}$. Then determine whether it is convergent/divergent. If it is convergent, compute its limit.
   (b) Find a closed-form formula for the $n$th term in the sequence $\left\{3, -4, \frac{16}{3}, -\frac{64}{9}, \ldots\right\}$. Then determine whether it is convergent/divergent. If it is convergent, compute its limit.
   (c) Find a closed-form formula for the $n$th term in the sequence $\{0.9, 0.09, 0.009, 0.0009, \ldots\}$. Then determine whether it is convergent/divergent. If it is convergent, compute its limit.
   (d) Find a closed-form formula for the $n$th term in the sequence $\{0.06, 0.006, 0.0006, 0.00006, \ldots\}$. Then determine whether it is convergent/divergent. If it is convergent, compute its limit.
   (e) Consider the sequence
   $$a_n = \frac{1}{(1\cdot 2)} + \frac{1}{(2\cdot 3)} + \frac{1}{(3\cdot 4)} + \cdots + \frac{1}{n\cdot(n+1)}. \tag{2}$$
   Compute and simplify the first four terms of $a_n$. Guess a general (closed-form) formula for $a_n$. Convince a study group mate that this is a correct formula.
   (f) Compute $\lim_{n\to\infty} a_n$, where $a_n$ is defined in line (2).

3. (Similar sequences to the WebAssign homework Sec 11.1) For a given sequence, use these strategies to determine its end behavior (Justify your answer or show work, but no formal proof is needed):
   (a) Apply l'Hopital's rule on a function related to a sequence when appropriate.
   (b) Apply the limit rules for sequences ($\pm$, product, quotient).
   (c) Apply limit rule by "taking the limit inside" of a *continuous* function when appropriate.
   (d) (NOT FOR WEEK 2 QUIZ - MOVED TO WEEK 3) Apply the squeeze theorem. For example, the sequence $a_n = (5n + 4)/(3n^2 - 2)$ converges to 0. Justify this fact by squeezing $a_n$ between 0 and another sequence of type $b_n = (\text{some constant})/n$ and using the Squeeze theorem. You may assume that $\lim_{n\to\infty} (\text{a constant})/n = 0$.
   (e) State the end behavior of a given geometric sequence.

4. (Sec 11.1 limit of simple continued fractions)

   (a) Compute positive integers $c_1, c_2, c_3, \ldots, c_m$ such that

$$\frac{33}{10} = [c_1, c_2, \ldots, c_m], \text{ where}$$

$$[c_1, c_2, \ldots, c_m] := c_1 + \cfrac{1}{c_2 + \cfrac{1}{\ddots + \cfrac{1}{c_m}}}$$

   (b) Compute positive integers $c_1, c_2, c_3, \ldots, c_m$ such that

$$\frac{29}{12} = [c_1, c_2, \ldots, c_m]$$

   (c) Compute the limit

$$[3, 3, 3, 3, 3, \ldots] = 3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{\ddots}}}}$$

5. (Sec 11.2 geometric series) A geometric series is convergent if and only if its terms converge to 0 (warning: this is not true in general for non-geometric series!) Examples:

   (a) Determine whether the (geometric) series

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \ldots$$

   converges (if so, please compute) or diverges.

   (b) Determine whether the (geometric) series

$$-3 + 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27}$$

   converges (if so, please compute) or diverges.

   (c) Determine whether the (geometric) series

$$0.9999\cdots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{10000} + \cdots = \sum_{n=1}^{\infty} \frac{9}{10^n}$$

   converges (if so, please compute) or diverges. Take a minute to consider whether your answer makes sense.

   (d) Do the same procedures for other numbers, for example, $0.2222\cdots$, $0.7777\cdots$, and $0.8888\cdots$.

   (e) Determine whether the (geometric) series

$$0.066666\cdots = \frac{6}{100} + \frac{6}{1000} + \frac{6}{10000} + \frac{6}{100000} + \cdots = \frac{6}{10} \sum_{n=1}^{\infty} \frac{1}{10^n}$$

   converges (if so, please compute) or diverges. Use your result to show that $\frac{1}{6} = 0.166666\ldots$. Then use another method (maybe long division) to verify this equality.

   (f) (FOR NEXT WEEK) Write $\frac{29}{12}$ as a (possibly repeating) decimal without using a calculator. Use geometric series to verify your answer.

**The following are questions that will be covered in future assessments. If you are finished with your quiz, please read and think about these questions.**

**UNGRADED Preview Problem 1** (Sec 11.1 monotonic convergence theorem) Give an example of a sequence satisfying the condition or explain why no such sequence exists:

- A monotonically increasing sequence that converges to 10.
- A monotonically increasing bounded sequence that does not converge (that is, diverges).
- A non-monotonic sequence that converges to 4.

**UNGRADED Preview Problem 2**

- Find *a* positive number $N$ such that, if $n > N$, then $\dfrac{1}{n^2-5} < \frac{1}{100}$.
- Find *a* positive number $N$ such that, $\dfrac{1}{n^2-5} < \frac{1}{7000}$ for all $n > N$.
- Then generalize this. Suppose I give you a positive number $\epsilon$. Find *a* positive number $N$ such that, if $n > N$, then $\dfrac{1}{n^2-5} < \epsilon$.

**UNGRADED Preview Problem 3** (Sec 11.1 limit of a sequence) The sequence $a_n = (2n+4)/(5n-8)$ converges to $2/5$. For any (small) number $\epsilon > 0$, choose $N$ so that if $n > N$, then $|2/5 - a_n| < \epsilon$.

(Sec 11.1 limit of a sequence) The sequence $a_n = a_n = (n^2+2)/(4n^2-1)$ converges to $1/4$. For any (small) number $\epsilon > 0$, find $N$ so that if $n > N$, then $|1/4 - a_n| < \epsilon$.

**UNGRADED Preview Problem 4** (Sec 11.2 limit definition for series)

(a) (Sec 11.2) Let $\{c_n\}$ be a sequence. What is a partial sum of $\{c_n\}$?

(b) (Sec 11.2) Let $\{c_n\}_{n=1}^{\infty}$ be a sequence. We say that the infinite series $\sum_{n=1}^{\infty} c_n$ is *convergent* if

________________________________________________________________. If the above blank is not true, then we say that $\sum_{n=1}^{\infty} c_n$ is *not convergent* or *divergent.*

**UNGRADED Preview Problem 5** (Sec 11.2) Divergence test: If a sequence does not converge to 0, then the corresponding infinite series diverges.

(a) Be able to apply the divergence test on infinite series whose terms do not converge to 0. Examples:

1. Determine whether the series
$$\sum_{n=1}^{\infty} \frac{2n}{3n-1} = 1 + \frac{4}{5} + \frac{3}{4} + \frac{8}{11} + \dots$$
converges or diverges.

(b) State whether the following statements are true or false. Explain briefly.

1. True or false? If $a_n$ does not converge to 0, then the series of $\sum_{n=1}^{\infty} a_n$ diverges.
2. True or false? If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.