Name (please print): _____ **Math 1152Q: Fall '17**

Week 3 Quiz, Friday, Sept 15, 2017

Please put all technology (phone, calculator, anything that has a computer) away.

Please sign the following academic integrity statement before turning in your quiz: "On my honor, I pledge that I will not give, receive, nor tolerate others' use of unauthorized aid in completing this work."

Signature: _____

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1. (Vocabulary) Do both problems.

   (a) Given a sequence $\{a_n\}$, what does $\lim_{n\to\infty} a_n = 4$ mean? Use the ϵ and N definition. Do *not* write 'limit', 'converge', 'approach', 'close to 4', etc.

   (b) Let $\{c_n\}_{n=1}^{\infty}$ be a sequence. If the infinite series $\sum_{n=1}^{\infty} c_n$ is *convergent*, we write $\sum_{n=1}^{\infty} c_n = L$ for some number L. This means that

   _____.

   (Hint: your answer *must* include the words 'partial sum'. You may use words like 'limit' and 'converge')

2. Do **ONE** of (a) or (b) below. *Circle* the problem you choose. Let ϵ be a positive number smaller than 1.

   (a) The sequence $a_n = \dfrac{n-1}{7n+4}$ converges to $1/7$. Choose N so that $|a_n - 1/7| < \epsilon$ whenever $n > N$. Show, as you did in Problems A, that this N works.

   (b) Give *a* positive number N such that, $\dfrac{1}{n^2-8} < \epsilon$ for all $n > N$. Show, as you did in Problems A, that this N works.

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3. Do exactly **ONE** of the questions (i) or (ii). Please circle your selection.

(i) (a) Find a formula for the n-th partial sum S_n of the series

$$\sum_{k=1}^{\infty} \frac{5}{k} - \frac{5}{k+3}.$$

(b) Evaluate $\lim_{n\to\infty} S_n$.

(c) Use part (b) to determine the sum of the series $\sum_{k=1}^{\infty} \frac{5}{k} - \frac{5}{k+3}$ or state that the series diverges.

(ii) (a) Find a formula for the n-th partial sum S_n of the series

$$\sum_{k=2}^{\infty} \frac{5}{k-1} - \frac{5}{k}.$$

(b) Evaluate $\lim_{n\to\infty} S_n$.

(c) Use part (b) to determine the sum of the series $\sum_{k=2}^{\infty} \frac{5}{k-1} - \frac{5}{k}$ or state that the series diverges.

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4. Do exactly **ONE** of the questions (a) or (b). Please circle your selection.

   Note: A *fraction* is an integer over an integer. Some examples of fractions are $\frac{24}{1}$ and $\frac{3}{7}$.

   (a) Use the geometric series formula to write the fraction for $0.1\overline{6} = 0.166666\ldots$.
   Show the work involved in setting up the geometric series. Make clear what your ratio is and include the first few terms of your series.

   (b) Use the geometric series formula to write the fraction for $4.\overline{9} = 4.999\ldots$.
   Show the work involved in setting up the geometric series. Make clear what your ratio is and include the first few terms of your series.

If you are finished with your quiz, do these exercises. There is no penalty for trying.

A Fill in the blanks with either the sign $\leq$ or $\geq$.

$$\frac{5n!}{2^n} \quad \text{———} \quad \left(\frac{1}{2}\right)^n \quad \text{for all } n \geq 1$$

$$\frac{n-1}{7n+4} \quad \text{———} \quad \frac{1}{7} \quad \text{for all } n \geq 1$$

$$\frac{n+1}{7n-4} \quad \text{———} \quad \frac{1}{7} \quad \text{for all } n \geq 1$$

B (Review) Sketch each function. Label the asymptote/s and zero/s of the graph. Hints:

- $f(x) = \frac{x-1}{7x+4}$ has a horizontal asymptote at $\frac{1}{7}$ because $\lim_{x\to\infty} f(x) = 1/7$.
- Note that $f(x) = \frac{x-1}{7x+4}$ is not defined for $x = -4/7$ and that $-4/7$ is not a zero of the numerator $x - 1$. This tells us that $f(x) = \frac{x-1}{7x+4}$ has a vertical asymptote at $x = -\frac{4}{7}$.
- The graph of $y = f(x)$ has a vertical asymptote $x = a$ if $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$.
- To figure out whether your graph approaches $+\infty$ or $-\infty$ from the right, plug in a number bigger than a and estimate whether it looks very large (positive) or very large (negative).

(a) $f(x) = \dfrac{x-1}{7x+4}$

(b) $g(x) = \dfrac{x+1}{7x-4}$

(c) $h(x) = \dfrac{1}{x+5}$

C (Review) By just looking at your sketches above, determine whether each of the following sequences is increasing or decreasing (or neither) for $n = 1, 2, 3, \ldots$.

(a) $\left\{\dfrac{n-1}{7n+4}\right\}_{n=1,2,3,\ldots}$ (b) $\left\{\dfrac{n+1}{7n-4}\right\}_{n=1}^{\infty}$ (c) $\left\{\dfrac{1}{n+5}\right\}_{n=1}^{\infty}$.

D Use your work above to quickly give a lower bound and an upper bound (*numbers*) for each of the following sequences.

(a) $\left\{\dfrac{n-1}{7n+4}\right\}_{n=1,2,3\ldots}$

(b) $\left\{\dfrac{n+1}{7n-4}\right\}_{n=1}^{\infty}$

(c) $\left\{\dfrac{1}{n+5}\right\}_{n=1}^{\infty}$