Name: ______________________________ **Math 1152Q: Fall '17**

Week 1 (Practice) Quiz

All technology (phone, calculator, anything that has a computer) should be stored in your bag.

1. (Reviewing vocabulary) Let f be a function and let a be a real number. According to Stewart's definition, f is called *continuous* at a if:

   1. $f(a)$ is ______________ ,

   2. ______________ exists, and

   3. $f(a) =$ ______________.

2. (Practice with sequences) For each part, (i) find a non-recursive formula for the nth term in the sequence. (ii) Describe the end behavior of this sequence. (iii) If the sequence is convergent, compute or guess its limit - explain (in words or using math symbols) how you get this number (no formal proof is needed).

   (a) $\{5, -5, 5, -5, 5, \dots\}$

   (b) $\{5, 8, 11, 14, 17, \dots\}$

   (c) $\left\{-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \dots\right\}$

3. (Memorizing vocabulary) (Note: $\mathbb{R}$ is the notation for the set of all real numbers.) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \frac{16x^2 - 8x - 6}{18x^2 - 6x + 8}.$$

We discussed in class on Wednesday that

$$\lim_{x\to\infty} f(x) = \frac{16}{18}. \tag{1}$$

Due to a theorem (Thm 3) explained in Sec 11.1 of Stewart, we can use equation (1) to show that the sequence $\{a_n\}$ defined by

$$a_n = \frac{16n^2 - 8n - 6}{18n^2 - 6n + 8}$$

also converges to $\frac{16}{18}$, or, in notation form,

$$\lim_{n\to\infty} a_n = \frac{16}{18}. \tag{2}$$

What does (2) mean (according to the definition of limit of a sequence)?

4. (Memorizing vocabulary) Let $\{b_n\}$ be a sequence. What does it mean to write $\lim_{n\to\infty} b_n = \infty$?

This sheet is not part of the quiz. If you are finished with the first sheet, please rip off this second sheet, and think about how to solve these problems. You can keep this second sheet. You can start with Problem 2 instead of Problem 1 if you like adding up continued fractions.

Problem 1

a. Compute the positive x such that

$$x = 1 + \frac{1}{x}.$$

Hint: use the quadratic formula.
Note: You solution is precisely the limit of $\frac{F(n+1)}{F(n)}$ as $n \to \infty$, where $F(n)$ denotes the n-th Fibonacci number. That is, your answer is the golden ratio.

b. Denote L to be the number you compute in part (a), the limit of $\frac{F(n+1)}{F(n)}$ as $n \to \infty$.

(i) Prove that $L^2 = \lim_{n\to\infty} \frac{F(n+2)}{F(n)}$. Hint: you may use the fact that L is the solution to the equation in part (a).

(ii) Optional: Show that $L^3 = \lim_{n\to\infty} \frac{F(n+3)}{F(n)}$.

(iii) Optional: Compute the limit of $\frac{F(n+4)}{F(n)}$ as $n \to \infty$.

Problem 2 (Ref: for a more formal way to approach the problem, see Sec 11.1 p92 page 706)

c. Compute the positive x such that

$$x = 2 + 1/x.$$

d. Let $a_1 = 2$. Simplify

$$a_2 := 2 + \frac{1}{2}$$

$$a_3 := 2 + \cfrac{1}{2 + \cfrac{1}{2}}$$

$$a_4 := 2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}$$

and keep going if you wish. The numerators and denominators you should be seeing are $1, 2, 5, 12, 29, 70, 169, 408, 985, 237$

Note: We will denote the three continued fractions above as $[2, 2]$, $[2, 2, 2]$, and $[2, 2, 2, 2]$ to indicate that we see two 2s, three 2s, and four 2s.

e. Consider the infinite continued fraction

$$[2, 2, 2, 2, 2, ...] \; = \; 2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}$$

This value is the limit of a_n as n approaches infinity, where $\{a_n\}$ is the sequence introduced in part (d). Compute this number.

Hint: The answer should be different from Problem 1, but the same technique applies.