

1. (Sec 11.1 monotone convergence theorem) Recall that *bounded* means bounded above and below.

(a) True or false? The sequence

$$\{\sin(n)\}_{n=1}^{\infty}$$

is bounded. Give upper and lower bounds (if true) or justification (if false).

(b) True or false? The sequence

$$\left\{ \frac{7n-8}{8n+3} \right\}_{n=1}^{\infty}$$

is bounded above. Give an upper bound (if true) or justification (if false). (Note: see WebAssign no. 7).

(c) True or false? The sequence

$$\left\{ \frac{7n-8}{8n+3} \right\}_{n=1}^{\infty}$$

is bounded below. Give a lower bound (if true) or justification (if false). (Note: see WebAssign no. 7).

(d) T or F? Every bounded sequence is convergent. Justify (if T) or give a counterexample (if F).

(e) T or F? There is a monotonically increasing sequence that converges to 10. Give an example (if T) or justify (if F).

(f) T or F? There exists a monotonically increasing bounded sequence that does not converge (that is, diverges). Provide an example (if T) or justify (if F).

(g) T or F? There is a non-monotonic sequence that converges to 4. Provide an example (if T) or justify (if F).

2. (Sec 11.1 proving convergence of a sequence)

(a) Give a positive number N such that, $\frac{1}{n^2-5} < \frac{1}{7000}$ for all $n > N$.

(b) I give you a positive number ϵ . Choose a positive number N such that, if $n > N$, then $\frac{1}{n^2-5} < \epsilon$.

(c) The sequence

$$\left\{ \frac{7n-8}{8n+3} \right\}_{n=1}^{\infty}$$

converges to $7/8$. For any number $\epsilon > 0$, find N so that $\left| \frac{7n-8}{8n+3} - \frac{7}{8} \right| < \epsilon$ as long as $n > N$.

(d) The sequence $a_n = (2n+4)/(5n-8)$ converges to $2/5$. For any number $\epsilon > 0$, choose N so that if $n > N$, then $|2/5 - a_n| < \epsilon$.

3. (Sec 11.1 sequences)

(a) Compute

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n.$$

(Hints: Note the indeterminate form of type 1^∞ . You can write $y = \left(1 + \frac{1}{x}\right)^x$ and $\ln(y) = x \left(1 + \frac{1}{x}\right)$. See l'Hopital handout pages 4-5 and WebAssign Sec 11.1 no. 5.)

(b) Compute

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^n \quad \text{and} \quad \lim_{n \rightarrow \infty} \left(1 + \frac{5}{4n}\right)^n \quad \text{if they exist.}$$

(c) Find the limit of the sequence

$$\left\{ \sqrt{5}, \sqrt{5\sqrt{5}}, \sqrt{5\sqrt{5\sqrt{5}}}, \dots \right\} \quad (\text{You can assume without explanation that it converges}).$$

4. (Sec 11.2 WebAssign homework)

(a) Determine whether each series is convergent or divergent by expressing s_n as a telescoping sum.

$$(i) \sum_{n=1}^{\infty} \frac{5}{n(n+1)}. \quad \text{For now, use the fact that: } \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$(ii) \sum_{n=2}^{\infty} \frac{8}{n^2 - 1}. \quad \text{For now, use the fact that: } \frac{2}{(n-1)(n+1)} = \frac{1}{n-1} - \frac{1}{n+1}$$

(b) T or F? If $\{a_n\}$ doesn't converge to 0, then the series of $\sum_{n=1}^{\infty} a_n$ diverges. If T, justify. If F, give a counterexample.

(c) T or F? If $\lim_{n \rightarrow \infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges. If T, justify. If F, give a counterexample.

(d) Determine whether the sequence $\left\{ \frac{-1 + n^2}{5n^2 + 100} \right\}_{n=1}^{\infty}$ Determine whether the series $\sum_{n=1}^{\infty} \frac{-1 + n^2}{5n^2 + 100}$ is convergent or divergent.

5. Apply squeeze theorem to answer the following questions.

(a) Determine whether

$$\left\{ \frac{1}{5 n! 2^n} \right\} \quad \text{converges or diverges.}$$

(Hint: squeeze this sequence between 0 and a geometric sequence. See also a related sequence on WebAssign Sec 11.1 no. 6).

(b) Consider the sequence $\left\{ \frac{\cos n}{\sqrt{n}} \right\}$ for $n \geq 1$. Use the squeeze theorem to show that this converges and find its limit. Hint: Find two sequences of the type $\frac{(\text{a constant})}{\sqrt{n}}$ that bound $\frac{\cos n}{\sqrt{n}}$. You may assume that $\lim_{n \rightarrow \infty} \frac{(\text{a constant})}{\sqrt{n}} = 0$.