

1.

$$\begin{aligned}
 0.\overline{38} &= 0.383838\cdots \\
 &= \frac{38}{100} + \frac{38}{100^2} + \frac{38}{100^3} + \cdots \\
 &= \frac{\frac{38}{100}}{1 - \frac{1}{100}} \\
 &= \frac{38}{99}
 \end{aligned}$$

Observation: put the repeating digits in the numerator and put N nine's in the denominator, where N is the number of repeating digits.

For example, $0.\overline{123} = \frac{123}{999}$

2.

$$\begin{aligned}
 1.\overline{38} &= 1 + 0.\overline{38} \\
 &= 1 + \frac{38}{99} \\
 &= \frac{99 + 38}{99} \\
 &= \frac{(100 - 1) + 38}{99} \\
 &= \frac{138 - 1}{99}
 \end{aligned}$$

Observation: put all digits minus nonrepeating digits in the numerator and put N nine's in the denominator, where N is the number of repeating digits.

For example, $3.\overline{652} = \frac{3652 - 3}{999}$

3.

$$\begin{aligned}
0.2\overline{74} &= 0.2 + 0.0747474\dots \\
&= \frac{2}{10} + \frac{74}{1000} + \frac{74}{1000 \times 100} + \frac{74}{1000 \times 100^2} + \dots \\
&= \frac{2}{10} + \frac{\frac{74}{1000}}{1 - \frac{1}{100}} \\
&= \frac{2}{10} + \frac{74}{990} \\
&= \frac{2 \times 99 + 74}{990} \\
&= \frac{2 \times (100 - 1) + 74}{990} \\
&= \frac{274 - 2}{990}
\end{aligned}$$

Observation: put all digits minus nonrepeating digits in the numerator and put N nine's and M zero's in the denominator, where N is the number of repeating digits and M is the number of nonrepeating digits (not including the integer part).

For example, $0.21\overline{95} = \frac{2195 - 21}{9900}$

4.

$$\begin{aligned}
1.2\overline{74} &= 1 + 0.2\overline{74} \\
&= 1 + \frac{274 - 2}{990} \\
&= \frac{990 + 274 - 2}{990} \\
&= \frac{(1000 - 10) + 274 - 2}{990} \\
&= \frac{1274 - 12}{990}
\end{aligned}$$

Observation: put all digits minus nonrepeating digits in the numerator and put N nine's and M zero's in the denominator, where N is the number of repeating digits and M is the number of nonrepeating digits (not including the integer part).

For example, $9.53\overline{4} = \frac{9534 - 953}{900}$