TASK 1.
Go to page 493. Read/skim the introduction.
Below, write down the question and solution to Example 1 (Sec 7.4, page 493). Make sure you understand every step. (Write the long division below without looking at the book.)

Motivation

Note that

$$\frac{1}{x-1}-\frac{1}{x+3}=\frac{(x+3)-(x-1)}{(x-1)(x+3)}=\frac{4}{x^2+2x-3}$$

The purpose of partial fractions is to reverse this process.

Partial Fractions	**Common Denominator**	Rational function
$\frac{1}{x-1}-\frac{1}{x+3}$	$\rightarrow$	$\frac{4}{x^2+2x-3}$
Easy to integrate	**Partial Fraction Decomposition**	Difficult to integrate
$\int\left(\frac{1}{x-1}-\frac{1}{x+3}\right)dx$	$\leftarrow$	$\int\frac{4}{x^2+2x-3}\,dx$

Definition

Let $f(x)=\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are **polynomials**.

- $f(x)$ is called a **rational function**.
- If **the degree** of $p(x)$ is **less than the degree** of $q(x)$, then $f(x)$ is called **proper**.

Procedure

Suppose $f(x)$ is a **proper rational function**.

1. Factor the Denominator.
2. Perform Partial Fraction Decomposition.
3. Clear Denominators by

   ____.
4. Solve for Unknowns by

   ____.

TASK 2a. Case 1 – (fill in the blank, copy from Sec 7.4, pg 494)

The denominator is a product of distinct linear factors

Suppose the denominator of a **proper rational function** can be written as

$$(a_1x+b_1)(a_2x+b_2) \quad (a_kx+b_k)$$

where no factor is repeated (and no factor is a constant multiple of another). Then there exists constants A_1, A_2, , and A_k such that

____.

Task 2b. Below, write down the question and solution to Example 3 (Sec 7.4, pg 495) but change the constant "a" to your favorite positive number.

Study (or skim) the solution to Example 2 (Sec 7.4, pg 494).
(Optional) TASK 3a: Below, write down the solution to Example 2 (Sec 7.4, pg 494).

TASK 3b. Follow book's Example 2 above (or a different strategy) to solve this similar problem:

Consider the function $f(x)=\dfrac{3x^2+7x-2}{x^3-x^2-2x}$.

a. Find the partial fraction decomposition for $f(x)$.

b. Evaluate $\int f(x)\,dx$.

Verify your answer with a computer. (Type "partial fraction" on Wolfram|Alpha).

TASK 4a. Case 2 – (fill the blank by copying the line labeled 7 from Sec 7.4, top of pg 496)
The denominator is a product of linear factors, some of which are repeated

Suppose the repeated linear factor $(ax+b)^m$ appears in the denominator of a **proper rational function**. The partial fraction decomposition has a partial fraction for each power of $(ax+b)$ up to and including the m th power. That is, there exists constants A_1, A_2, , and A_m such that the partial fraction decomposition contains the sum

____.

TASK 4b. Finish the following incomplete solution:

Consider the function $f(x)=\dfrac{5x^2-3x+2}{x^3-2x^2}$.

1. Find the partial fraction decomposition for $f(x)$.

Solution: We use the above Case 2 theorem to know that we can write

$$\frac{5x^2-3x+2}{x^3-2x^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-2}$$

From here, follow the same strategy of Example 4 (or another method) to compute A, B, and C. Verify your work - type on Wolfram|Alpha: “partial fraction ((5x^2 - 3x + 2)/(x^3 - 2x^2))”

2. Evaluate $\int f(x)\,dx$.

Task 4a. Case 3 – (fill in the blank by copying from pg 497)

The denominator contains irreducible quadratic factors, none of which is repeated

Suppose the denominator of a **proper rational function** has the factor ax^2+bx+c, where $b^2-4ac<0$. There exists constants A and B such that the partial fraction decomposition contains a term of the form

____.

Task 4b. Write down the integral to Example 5 (pg 497). Copy down the solution only as far as computing A, B, C. Show your work.

(Optional: finish computing the integral)

Note: Case I, II, III are the most common types that will be useful for differential equations. (You can read about case IV (pg 499) if you're interested, but I won't cover it.)