 Trigonometric Integrals Name: ________________________

Integrating Powers of tangent

Example (in class):
Evaluate $\int \tan^4 x\, dx$.

Strategy Tangent-1

For $k \geq 0$, evaluate $\int \tan^{k+2} x\, dx$.

$\int \tan^{k+2} x\, dx =$ ____________________ Separate one __________ factor.

$=$ ____________________ Use the identity ____________________.

$=$ ____________________ + ____________________

Let $u =$ __________ Use Strategy T-1

Task 1 (three parts): Review u-substitution and chain rule/ quotient rule

Review: Evaluate the following indefinite integral using u-substitution with u=cos x.

Anti-Derivative

$$\int \tan x \, dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

1a. Check your work or learn how to do this by looking at the book's solution: Sec 5.5 Example 6, pg 415-416. Write your work here.

Knowing the derivatives for cosine and sine functions, use chain rule/ quotient rule to compute:

Derivative $\frac{d}{dx}(\tan x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$

1b. Check your work or learn how to do this by looking at the book's solution: Sec 3.3 pg 193. Write your computation work here.

Given the derivatives for cosine and sine functions, use chain rule or quotient rule to compute

Derivative $\frac{d}{dx}(\sec x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$

1c. Sec 3.3 pg 193 gives you the answer but not explanation. Check your answer or learn how to do it by watching this Khan academy video. (Note: the video uses quotient rule but you may find the chain rule to be less work in this case): https://www.khanacademy.org/math/ap-calculus-ab/ab-derivative-rules/ab-diff-trig-func/v/derivatives-of-secx-and-cscx Write your work here.

Integrating Products of tangent and secant (power of tangent is odd or power of secant is even) where you can use u-substitution

Let m and n be integers. Evaluate $\int \tan^m x \sec^n x \, dx$.

Strategy: The power of tangent is odd (see Example 6)

$m = 2k + 1 > 0$

$$\begin{aligned}\int \tan^m x \sec^n x \, dx &= \int \left(\tan^2 x\right)^k \sec^{n-1} x \sec x \tan x \, dx \\ &= \int \left(\sec^2 x - 1\right)^k \sec^{n-1} x \sec x \tan x \, dx \\ &= \int \left(u^2 - 1\right)^k u^{n-1} \, du\end{aligned}$$

Strategy: The power of secant is even (see Example 5)

$n = 2k > 0$

$$\begin{aligned}\int \tan^m x \sec^n x \, dx &= \int \tan^m x \left(\sec^2 x\right)^{k-1} \sec^2 x \, dx \\ &= \int \tan^m x \left(1 + \tan^2 x\right)^{k-1} \sec^2 x \, dx \\ &= \int u^m \left(1 + u^2\right)^{k-1} \, du\end{aligned}$$

Task 2 (two parts): Examples.

Strategy: The power of tangent is odd (Example 6)

2a. Below, please evaluate the indefinite integral given in Sec 7.2, Example 6, pg 482. Try to first solve it without looking at the book's solution.

Strategy: The power of secant is even (Example 5)

2b. Below, please evaluate the indefinite integral given in Sec 7.2, Example 5, pg 481. Try to first solve it without looking at the book's solution.

Note: If the power of tangent is odd and the power of secant is even, either strategy can be used. But what if neither is true? Then the computation is slightly longer. Will learn a strategy for this later (reference: notes Sec 7.1 part 2).

Task 3: Evaluate the following using u-substitution. Follow book's solution Sec 7.2 pg 483.

Anti-Derivative

$$\int \sec x \, dx = \underline{\hspace{10cm}}.$$

Write your work here: