Trigonometric Integrals

Integrating Powers of sine and cosine

Example:

Evaluate $\int \cos^5 x \, dx$.

Strategy Cosine-Odd

For $k \geq 0$, evaluate $\int \cos^{2k+1} x \, dx$.

$\int \cos^{2k+1} x \, dx =$ ____________________ Separate one __________ factor.

$=$ ____________________ Convert $\cos^{2k} x$ to __________.

$=$ ____________________ Use the identity ____________________.

$=$ ____________________ Let $u =$ __________, then $du =$ __________.

Strategy Sine-Odd

For $k \geq 0$, evaluate $\int \sin^{2k+1} x \, dx$.

$\int \sin^{2k+1} x \, dx =$ ____________________ Separate one __________ factor.

$=$ ____________________ Convert $\sin^{2k} x$ to __________.

$=$ ____________________ Use the identity ____________________.

$=$ ____________________ Let $u =$ __________, then $du =$ __________.

Example: (Example 4 pg 480)

Evaluate $\int \sin^4 x \, dx$.

Strategy Cosine-Even

For $k \geq 0$, evaluate $\int \cos^{2k} x \, dx$.

$\int \cos^{2k} x \, dx =$ ____________________ Convert $\cos^{2k} x$ to __________.

$=$ ____________________ Use the formula ____________________.

$=$ ____________________ Foil out and use Strategy Cos-Odd and Cos-Even

Strategy Sine-Even

For $k \geq 0$, evaluate $\int \sin^{2k} x \, dx$.

$\int \sin^{2k} x \, dx =$ ____________________ Convert $\sin^{2k} x$ to __________.

$=$ ____________________ Use the formula ____________________.

$=$ ____________________ Foil out and use Strategy Cos-Odd and Cos-Even

Integrating Products of sine and cosine

Let m and n be integers. Evaluate $\int \sin^m x \cos^n x\, dx$.

Strategy The power of cosine is odd (same strategy as Cosine-odd pg 1)

$n = 2k+1 > 0$

$$\begin{aligned}\int \sin^m x \cos^n x\, dx &= \int \sin^m x \left(\cos^2 x\right)^k \cos x\, dx \\ &= \int \sin^m x \left(1-\sin^2 x\right)^k \cos x\, dx \\ &= \int u^m \left(1-u^2\right)^k\, du\end{aligned}$$

Strategy The power of sine is odd (same strategy as Sine-odd pg 1)

$m = 2k+1 > 0$

$$\begin{aligned}\int \sin^m x \cos^n x\, dx &= \int \left(\sin^2 x\right)^k \cos^n x \sin x\, dx \\ &= \int \left(1-\cos^2 x\right)^k \cos^n x \sin x\, dx \\ &= -\int \left(1-u^2\right)^k u^n\, du\end{aligned}$$

If the powers of both sine and cosine are odd, either strategy can be used.

Strategy 3-2 The powers of both sine and cosine are even (Combine Cosine-even and Sine-even strategy pg 2)

$n = 2k \geq 0$ and $m = 2h \geq 0$

$$\begin{aligned}\int \sin^m x \cos^n x\, dx &= \int \left(\sin^2 x\right)^h \left(\cos^2 x\right)^k\, dx \\ &= \int \left(\frac{1-\cos 2x}{2}\right)^h \left(\frac{1+\cos 2x}{2}\right)^k\, dx\end{aligned}$$

Foil out and use Strategy Cosine-Odd and Cosine-Even