Name: ___________________ **Integration by Parts** Notes

Recall

Reverse **Chain Rule** to get **Substitution Rule**.

$$\frac{d}{dx}\left[f\left(g\left(x\right)\right)\right]=f'\left(g\left(x\right)\right)g'\left(x\right)$$

$$\int f'\left(g\left(x\right)\right)g'\left(x\right)\,dx=f\left(g\left(x\right)\right)+C$$

Let $u=g\left(x\right)$, then $du=g'\left(x\right)dx$. Thus

$$\int f'\left(g\left(x\right)\right)g'\left(x\right)\,dx=\int f'\left(u\right)\,du=f\left(u\right)+C=f\left(g\left(x\right)\right)+C.$$

Integration by Parts

Reverse **Product Rule** to get **Integration by Parts**.

$$\frac{d}{dx}\left[u\left(x\right)v\left(x\right)\right]=u'\left(x\right)v\left(x\right)+u\left(x\right)v'\left(x\right)$$

$$\int u'\left(x\right)v\left(x\right)\,dx+\int u\left(x\right)v'\left(x\right)\,dx=u\left(x\right)v\left(x\right)+C$$

$$\int u\left(x\right)v'\left(x\right)\,dx=u\left(x\right)v\left(x\right)-\int u'\left(x\right)v\left(x\right)\,dx$$

Since $\frac{dv}{dx}=v'\left(x\right)$ and $\frac{du}{dx}=u'\left(x\right)$, we can obtain

$$\int u\,dv=uv-\int v\,du.$$

Integration by Parts

Suppose that u and v are differentiable functions. Then,

$$\int u\,dv=uv-\int v\,du.$$

Integration by Parts is an integration technique for evaluating integrals of **product of functions**.

Integration by Parts

To use Integration by Parts, one should

- Choose u and dv. Note: dv should be **easy to integrate**.
- Evaluate du and v.
- Apply the formula.

Example:
Evaluate $\int xe^x \, dx$.

Integration by Parts for Definite Integrals

Let u and v be differentiable. Then,

$$\int_a^b u \, dv = uv\Big|_a^b - \int_a^b v \, du.$$

TASK 1: First attempt on your own. Then follow pg 473, Sec 7.1 Ex 2 to evaluate the indefinite integral. Compute the definite integral on your own. Check your answer with WolframAlpha.

Evaluate $\int_1^e \ln x \, dx$.

Integration by Parts

Recall

Integration by Parts

Suppose that u and v are differentiable functions. Then,

$$\int u\, dv = uv - \int v\, du.$$

Repeated Use of Integration by Parts

[Type 1] Use Integration by Parts AGAIN.

TASK 2: First attempt on your own. This requires multiple applications of integration by parts. Then follow the solution given on pg 474 Sec 7.1 Example 3.

Evaluate $\int x^2 e^x\, dx$.

Integration by Parts

[Type 2] Use Integration by Parts AGAIN + MERGE.

TASK 3: First attempt on your own (it does take multiple steps using Calc II methods).
Then follow the solution given on pg 474 Sec 7.1 Ex 4.
Evaluate $\int e^x \sin x \, dx$.

Products of tangent (even power) and secant (odd power) - you cannot use u-substitution

TASK 4: Evaluate $\int \tan^2 x \sec x \, dx$.

Instruction:

1. First, use the identity $\tan^2 \theta + \_\_\_\_ = \sec^2 \theta$ to get rid of the tangent.

2. Evaluate the antiderivative of (sec^3 x) on your own and by copying pg 483 Sec 7.2 Ex 8.

3. You've already evaluated the antiderivative for (sec x) in your last reading homework: https://egunawan.github.io/fall17/notes/notes7_2part2.pdf

Look this up at the top of page 483 (Don't memorize this antiderivative! But be able to evaluate this using u-substitution after receiving hints for what to do).
(Verify your solution by looking at the answer key to Learning Activity 7.2 part 2: https://egunawan.github.io/fall17/notes/LA7_2part2key.pdf)