Section 11.8 | Reading HW due Week 8, Mon, Oct 16, 2017 | MATH 1152Q

Name:________________________ **Power Series** Notes

Recall

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + r^3 + \cdots \ \ldots = \frac{1}{1-r} \text{ provided } |r| < 1.$$

Replace the real number r by the variable x, then (fill in the blank, eq. 2 pg 746)

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The infinite series is a **power series**.

Definition Power Series

A **power series** has the general form (follow page 747, equation 3)

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where a and c_k are real numbers and x is a variable.

- The c_k's are the **coefficients** of the power series and a is the **center** of the power series.
- The set of values of x for which the series converges is the **interval of convergence**.
- The **radius of convergence** of the power series, denoted R, is the distance from the center of the series to the boundary of the interval of convergence.

The sum of the series is a function

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$$

whose domain is the set of all x for which the series converges.

Strategy **Find the Interval and Radius of Convergence**

For **ALL** power series,

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Let $r = \lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{c_{k+1}(x-a)^{k+1}}{c_k(x-a)^k}\right| = \lim_{k\to\infty}\left|\frac{c_{k+1}}{c_k}\right||x-a|$.

We want the power series to be convergent. That is, we want $r<1$.

1. If $\lim_{k\to\infty}\left|\frac{c_{k+1}}{c_k}\right| = 0$, then $r = |x-a|\lim_{k\to\infty}\left|\frac{c_{k+1}}{c_k}\right| = 0 < 1$ for all x.
   Thus the Interval of Convergence is $(-\infty,\infty)$ and the Radius of Convergence is ∞.

2. If $\lim_{k\to\infty}\left|\frac{c_{k+1}}{c_k}\right| = \infty$, then $r = \lim_{k\to\infty}\left|\frac{c_{k+1}}{c_k}\right||x-a| < 1$ if and only if $|x-a| = 0$.
   Thus the Radius of Convergence is 0.

3. If $\lim_{k\to\infty}\left|\frac{c_{k+1}}{c_k}\right| = L$, then $r = |x-a|\lim_{k\to\infty}\left|\frac{c_{k+1}}{c_k}\right| = L|x-a| < 1$ if and only if $|x-a| < \frac{1}{L}$.
   Thus the Radius of Convergence is $\frac{1}{L}$.

Since the Ratio Test is inconclusive when $r=1$, we need to check the **End Points** and determine the Interval of Convergence.

Theorem **Convergence of Power Series**

A power series $\sum_{k=0}^{\infty} c_k(x-a)^k$ centered at a converges in one of three ways:

1. The series converges absolutely for all x, in which case the interval of convergence is $(-\infty,\infty)$ and the radius of convergence is $R=\infty$.
2. There is a real number $R>0$ such that the series converges absolutely for $|x-a|<R$ and diverges for $|x-a|>R$, in which case the radius of convergence is R.
3. The series converges only at $x=a$, in which case the radius of convergence is $R=0$.

Example (optional):

Find the interval and radius of convergence for the power series $\sum_{k=0}^{\infty} \frac{x^k}{k!}$.

Task 1. Practice (copy or verify with the book's Example 1, pg 747. See the table at the bottom of pg 749):

Find the interval and radius of convergence for the power series $\sum_{k=1}^{\infty} k!x^k$.

Task 2. Practice (verify with the book's Example 2, pg 747. See the table at bottom of pg 749):

Find the interval and radius of convergence for the power series $\sum_{k=1}^{\infty} \frac{(x-2)^k}{k}$.

Strategy

If the given power series is a **Geometric Series**, then

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Task 3. Practice (Please show work. Will be discussed in class):

Find the interval and radius of convergence for the power series $\sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^k}{4^k}$.

Solution: The series converges if and only if $-2 < x < 6$ by the geometric series test. So the interval of convergence is $I = (-2, 6)$ and the radius of convergence is $R = 4$.