Strategy

Suppose $\sum_{n=1}^{\infty} a_n$ is an infinite series.

The Geometric Series → when $\sum_{n=1}^{\infty} a_n$ has the form $\sum_{n=1}^{\infty} cr^n$.

If $|r| \geq 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.

If $|r| < 1$, then $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} a_n = \frac{a_1}{1-r}$.

The Divergence Test

If $\lim_{n\to\infty} |a_n| \neq 0$, then $\lim_{n\to\infty} a_n \neq 0$, so $\sum_{n=1}^{\infty} a_n$ diverges.

If $\lim_{n\to\infty} |a_n| = 0$, then the Divergence Test is **inconclusive.**

Consider $\sum_{n=1}^{\infty} |a_n|$.

The *p*-Series → when $\sum_{n=1}^{\infty} |a_n|$ has the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$.

If $p > 1$, then $\sum_{n=1}^{\infty} |a_n|$ converges.

If $p \leq 1$, then $\sum_{n=1}^{\infty} |a_n|$ diverges. **Go to Alternating Series Test.**

The Ratio Test → when a_n involves factorials or powers. Consider $r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$.

If $0 \leq r < 1$, then $\sum_{n=1}^{\infty} |a_n|$ converges

If $r > 1$, then $\sum_{n=1}^{\infty} |a_n|$ diverges. **Go to Alternating Series Test.**

If $r = 1$, then the Ratio Test is **inconclusive.**

Strategy

The Limit Comparison Test → when $|a_n|$ involves dominant terms. Consider $\lim_{n\to\infty} \frac{|a_n|}{b_n} = L$.

If $0 < L < \infty$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} |a_n|$ converges.

If $0 < L < \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} |a_n|$ diverges. **Go to Alternating Series Test.**

If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} |a_n|$ converges.

If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ diverges, then the Limit Comparison Test is **inconclusive.**

If $L = \infty$ and $\sum_{n=1}^{\infty} b_n$ converges, then the Limit Comparison Test is **inconclusive.**

If $L = \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} |a_n|$ diverges. **Go to Alternating Series Test.**

The Comparison Test

If $0 < |a_n| \le b_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} |a_n|$ converges.

If $0 < |a_n| \le b_n$ and $\sum_{n=1}^{\infty} b_n$ diverges, then the Comparison Test is **inconclusive.**

If $0 < b_n \le |a_n|$ and $\sum_{n=1}^{\infty} b_n$ converges, then the Comparison Test is **inconclusive.**

If $0 < b_n \le |a_n|$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} |a_n|$ diverges. **Go to Alternating Series Test.**

The Integral Test → when $|a_n|$ looks easy to integrate. Consider $f(x) = |a_x|$ for $x \ge 1$.
$f(x)$ need to be positive, continuous and decreasing for $x \ge 1$.

If $\int_1^{\infty} f(x)\, dx$ converges, then $\sum_{n=1}^{\infty} |a_n|$ converges.

If $\int_1^{\infty} f(x)\, dx$ diverges, then $\sum_{n=1}^{\infty} |a_n|$ diverges. **Go to Alternating Series Test.**

Strategy

Consider $\sum_{n=1}^{\infty} a_n$.

The Alternating Series Test

If $\{|a_n|\}$ is decreasing, then $\sum_{n=1}^{\infty} a_n$ converges, so $\sum_{n=1}^{\infty} a_n$ converges conditionally.