Growth Rates

Definition **Growth Rates of Functions (as x approaches infinite)**

Suppose f and g are functions with $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$. Then

- f **grows faster than** g as $x \to \infty$ if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$.
- f and g **have comparable growth rates** if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = M$, where $0 < M < \infty$.

Theorem **Asymptotic Hierarchy**

Let $f \ll g$ mean that g grows faster than f as $x \to \infty$. Then

$$c \ll (\ln x)^q \ll x^p \ll a^x \ll x! \ll x^x$$

$$\lim_{n\to\infty} \sqrt[n]{c} \qquad \lim_{n\to\infty} \sqrt[n]{(\ln n)^q} \qquad \lim_{n\to\infty} \sqrt[n]{n^p} \qquad \lim_{n\to\infty} \sqrt[n]{a^n} \qquad \lim_{n\to\infty} \sqrt[n]{n!} \qquad \lim_{n\to\infty} \sqrt[n]{n^n}$$

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Example:

Use the **Root Test** to determine whether the series $\sum_{k=2}^{\infty} \frac{3^k \ln k}{2^k k^5}$ converge.