Remainders in Alternating Series (pg 735)

Theorem Remainder in Alternating Series/ Alternating Series Estimation Theorem

Let $R_n = S - S_n$ be the **remainder** in approximating the value of a convergent alternating series

$\sum_{n=1}^{\infty} (-1)^{n+1} b_n$ by the sum of its first n terms. Then

$$|R_n| \leq b_{n+1}.$$

In other words, the remainder is less than or equal to the magnitude of the first neglected term.

Example:
Determine how many terms of the following convergent series must be summed to be sure that the remainder is less than 10^{-4}?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + L \quad \ldots \quad = \ln 2$$

Fun fact: Use the **Alternating Series Estimation Theorem** to prove that e is irrational.