The Comparison Test

Go to page 727 Sec 11.4. Skim the two paragraphs.
Task 1. Fill in the blanks by copying from the bottom of this page.

Theorem The Comparison Test

Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are infinite series with **positive** terms.

- If $a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ **converges**, then ______________________ .
- If $a_n \ge b_n$ and $\sum_{n=1}^{\infty} b_n$ **diverges**, then ______________________ .

Task 2. Try to fill in the blanks below (it's OK if you write 'not sure')

- If $a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ **diverges**, then ________________________________ .
- If $a_n \ge b_n$ and $\sum_{n=1}^{\infty} b_n$ **converges**, then ________________________________ .

Task 3. Example:
Type "sum n/(n^2 + 1), n=1 to infty" into WolframAlpha and write down the Convergence tests that WolframAlpha tried to use (this should appear right after the result):

____

____

____

Task 4a. Don't try to solve yet. We want to use the **Comparison Test** to determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ converges. Below, please compare the terms of this series with the terms of the harmonic series (Sec 11.2 Example 9 on page 713). In particular, write down which series has larger terms and why.

Task 4b. Can you use the Divergence Test from Sec 11.2 to determine whether the series above converges or diverges? Explain.

The Limit Comparison Test

Task 5. First go to page 729 Sec 11.4. Fill in the blanks.

Theorem The Limit Comparison Test

Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are infinite series with **positive** terms. Let $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

- If L is a positive *number*, then (fill in the blank by copying the book Sec 11.4 pg 729)

  ____.

- If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ **converges**, then ___________________________________

  (fill in the blank by copying the question of Sec 11.4 p40 on page 732).

- If L is replaced with the infinity symbol and $\sum_{n=1}^{\infty} b_n$ **diverges**, then

  ____

  (fill in the blank by copying the question of Sec 11.4 p41 on page 732).

Task 6. Try to fill in the blanks below (it's OK if you write 'not sure')

- If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ **diverges**, then __________________________________.
- If $L = \infty$ and $\sum_{n=1}^{\infty} b_n$ **converges**, then __________________________________.

Task 7. Go to page 728 Sec 11.4. The paragraph (right before Example 1) tells you how to fill in the blanks.

p-series test

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is

convergent if ________________________ and divergent if ________________________.

Example: Consider the series the series $\sum_{n=1}^{\infty} \frac{n^4 - 2n^2 + 3}{2n^6 - n + 5}$. We will compare this series with a p-series $\sum_{n=1}^{\infty} b_n$ where p=2 (this p-series converges by the p-series test above).

Task 8a. Below, compute L where $\lim_{n \to \infty} \frac{a_n}{b_n} = L$.

Task 8a. Using the **Limit Comparison Test** and your computation above, determine whether the series $\sum_{n=1}^{\infty} \frac{n^4 - 2n^2 + 3}{2n^6 - n + 5}$ converges.