The Integral Test and Estimates of Sums

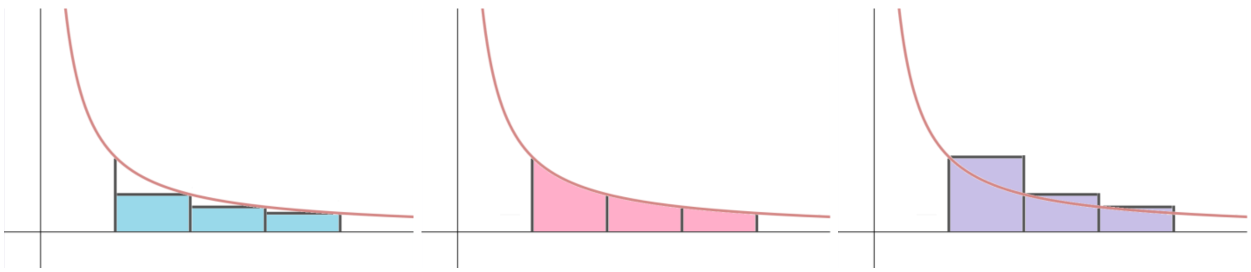
Estimating the Sum of a Series (pg 723)

We want to find an approximation to a convergent series $\sum a_n = S$. Any partial sum S_n is an approximation to S since $\lim_{n\to\infty} S_n = S$. But how good is such an approximation?

To find out, we need to estimate the size of the remainder

$$R_n = S - S_n = a_{n+1} + a_{n+2} + a_{n+3} + \quad \ldots$$

The remainder R_n is the error made when S_n is used as an approximation to S.



Remainder Estimate for the Integral Test (*)

Suppose $f(k) = a_k$, where f is a continuous, positive, decreasing function for $x \geq n$ and $\sum a_n$ is convergent. If $R_n = S - S_n$, then

$$\underline{\hspace{4cm}} \leq R_n \leq \underline{\hspace{4cm}}.$$

Example 5 pg 723:

Consider the approximation of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ by using the sum of the first 10 terms.

a. Estimate the error.
b. How many terms are required to ensure that the sum is accurate to within 0.0005?

If we add S_n to each side of the inequalities in the previous result (*), we get

______________________ $\leq S \leq$ ______________________. (**)

Note: (**) gives a better estimate to the sum of the series than the partial sum S_n does.

Example 6:

Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. Given $S_{10} \approx 1.197532$, estimate the sum of the series.