Telescoping Series

Example:

Consider the infinite series $\sum_{k=1}^{\infty}\left(\sqrt{k+1}-\sqrt{k}\right)$.

(1) Find a formula for the n-th term of the sequence of **partial sums** $\{S_n\}$.

(2) Evaluate $\lim_{n\to\infty} S_n$ to obtain the value of the series or state that the series diverges.

Divergence Test

Theorem

If the series $\sum_{k=1}^{\infty} a_k$ is convergent, then $\lim_{k\to\infty} a_k =$ ______________________.

With any series $\sum a_n$ we associate two sequences:

- the sequence $\{a_n\}$ of its **terms** and
- the sequence $\{S_n\}$ of its **partial sums**.

If $\sum a_n$ is convergent to S, then

$\lim_{n\to\infty} S_n =$ ______________________ and $\lim_{n\to\infty} a_n =$ ______________________.

Divergence Test

If $\lim_{k\to\infty} a_k \neq 0$, then the series $\sum_{k=1}^{\infty} a_k$ is ______________________.

If ______________________, then **the test is inconclusive**. The test cannot be used to determine convergence.

Theorem Harmonic Series

The **harmonic series** $\sum_{k=1}^{\infty} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ ______________________

However, ____.

Example:

Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{2k+1}$ diverges or state that the test you used is inconclusive.

Example:

Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ diverges or state that the test you used is inconclusive.

Properties of Convergent Series

Theorem

If $\sum a_k$ and $\sum b_k$ are convergent series, then

- Then the series $\sum ca_k$ **converges** and

$$\sum ca_k = \underline{\hspace{6cm}}.$$

- Then the series $\sum (a_k \pm b_k)$ **converges** and

$$\sum (a_k \pm b_k) = \underline{\hspace{6cm}}.$$

- If M is a positive integer, then $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=M}^{\infty} a_k$ both converge or both diverge.

Note

Whether a series converges does not depend on a finite number of terms added to or removed from the series. However, the value of a convergent series does change if nonzero terms are added or deleted.