Series

Telescoping Series

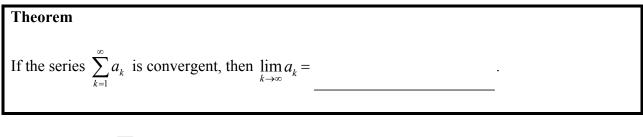
Example:

Consider the infinite series $\sum_{k=1}^{\infty} \left(\sqrt{k+1} - \sqrt{k} \right)$.

(1) Find a formula for the *n*-th term of the sequence of **partial sums** $\{S_n\}$.

(2) Evaluate $\lim_{n\to\infty} S_n$ to obtain the value of the series or state that the series diverges.

Divergence Test



With any series $\sum a_n$ we associate two sequences:

- the sequence $\{a_n\}$ of its **terms** and
- the sequence $\{S_n\}$ of its **partial sums**.

If $\sum a_n$ is convergent to S, then

 $\lim_{n \to \infty} S_n = \frac{1}{\sum_{n \to \infty} and \lim_{n \to \infty} a_n} = \frac{1}{\sum_{n \to \infty} and \lim_{n \to \infty} and \lim_{n \to \infty} a_n} = \frac{1}{\sum_{n \to \infty} and \lim_{n \to \infty} and \lim_{$

Divergence Test
If
$$\lim_{k \to \infty} a_k \neq 0$$
, then the series $\sum_{k=1}^{\infty} a_k$ is ______.

If

, then the test is inconclusive. The test cannot be used to

determine convergence.

Theorem Harmonic Series
The harmonic series
$$\sum_{k=1}^{\infty} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + L$$

However,

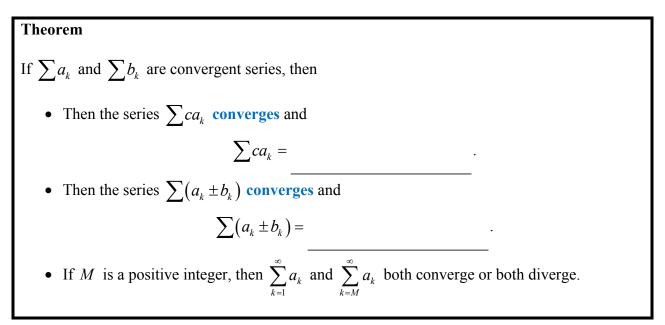
Example:

Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{2k+1}$ diverges or state that the test you used is inconclusive.

Example:

Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{k^2 + 1}$ diverges or state that the test you used is inconclusive.

Properties of Convergent Series



Note

Whether a series converges does not depend on a finite number of terms added to or removed from the series. However, the value of a convergent series does change if nonzero terms are added or deleted.