Name: ________________ **Taylor and Maclaurin Series** Notes

Recall (fill in the blank, see middle of pg 761)

The n-th Taylor Polynomial centered at a is

____.

Remainder in a Taylor Polynomial

Taylor polynomials provide good approximations to functions near a specific point, but how good are the approximations?

Let $R_n(x) = f(x) - T_n(x)$, then $R_n(x)$ is called the remainder of the Taylor series.

(Copy from pg 962) Theorem Taylor's Inequality

Suppose there exists a number M such that

$$\left|f^{(n+1)}(x)\right| \le M \text{ for } |x-a| \le d,$$

then the remainder $R_n(x)$ of the Taylor series satisfies

$|R_n(x)| \le$ ______________________ for ______________________ .

Complete the Example: Consider the function $f(x) = \sqrt[3]{x}$.

a. Find the **Taylor polynomials of order 2** centered at $x = 8$ for $f(x)$.
b. How accurate is this approximation when $7 \le x \le 9$. Follow Sec 11.11 Example 1 pg 775.

(Fill in each blank) Commonly used Maclaurin Series

- $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$.
- $\dfrac{1}{1+x} =$ ____________________________ for ____________________ .
- $\ln(1-x) = -\sum_{n=0}^{\infty} \dfrac{x^n}{n}$ for $-1 \le x < 1$.
  - $\sum_{n=0}^{\infty} \dfrac{1}{n2^n} =$ __________ .
- $\ln(1+x) =$ ____________________________ for ____________________ .
- $\tan^{-1} x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2n+1}$ for $-1 \le x \le 1$.
  - $\sum_{n=0}^{\infty} \dfrac{(-1)^n}{2n+1} =$ __________ .
- $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$ for $-\infty < x < \infty$.
  - $\sum_{n=0}^{\infty} \dfrac{1}{n!} =$ __________ .
- $\sin x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}$ for $-\infty < x < \infty$.
  - $\sum_{n=0}^{\infty} \dfrac{(-1)^n \pi^{2n+1}}{(2n+1)!} =$ __________ .
- $\cos x =$ ____________________________ for ____________________ .