Areas in Polar Coordinates

Polar Coordinates

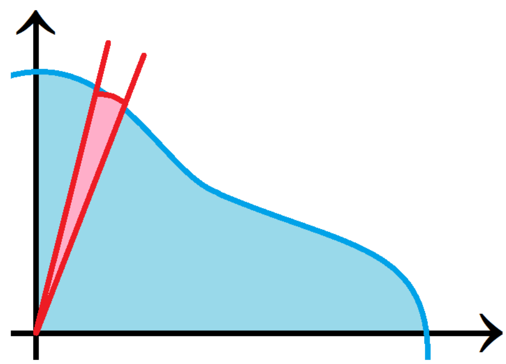
The area of the sector of a circle swept out by a 90 degree angle with radius r is ____________.

The area of the sector of a circle swept out by an angle θ with radius r is _________________.

The objective is to find the area of the region R bounded by the graph of $r = f(\theta)$ between the two rays $\theta = \alpha$ and $\theta = \beta$. Here, we assume that f is **continuous** and **nonnegative** on $[\alpha, \beta]$.

The area of R is found by slicing the region in slices.

Let $\Delta\theta_k = \theta_k - \theta_{k-1}$ and θ_k^* be any point of the interval $[\theta_{k-1}, \theta_k]$ for $k = 1, 2, \ldots, n$.



The k th slice is approximated by the sector of a circle swept out by an angle $\Delta\theta_k$ with radius $f(\theta_k^*)$.

Therefore, the area of the k th slice is approximately _______________________.

To find the area of R, we sum the areas of these slices and take more sectors ($n \to \infty$).

The exact area is given by

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2}\left[f\left(\theta_k^*\right)\right]^2 \Delta\theta_k = \underline{\hspace{6cm}}.$$

Note the similarity between the formulas. It is helpful to think of the area as being swept out by a rotating ray through the pole that starts with angle α and ends with angle β.

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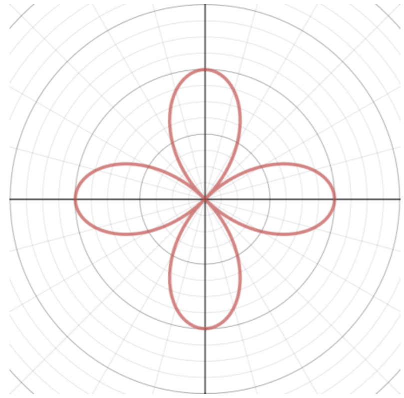
Example:
Find the area enclosed by one loop of the four-leaved rose $r = 4\cos 2\theta$.



Example:
Find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.