Tangent Lines

Suppose $h(t)$ and $k(t)$ are differentiable functions of t. Consider the parametric curve

$$\begin{cases} x = h(t) \\ y = k(t) \end{cases}, \text{ where } y \text{ is also a differentiable function of } x.$$

First Derivative

$$\frac{dy}{dx} = \underline{\hspace{3cm}} \qquad \text{if } \underline{\hspace{3cm}}$$

- The curve has a **horizontal tangent** when ______________________________ .
- The curve has a **vertical tangent** when ______________________________ .

Example: Consider the parametric curve (**cycloid**) $\Gamma : \begin{cases} x = 2(\theta - \sin\theta) \\ y = 2(1 - \cos\theta) \end{cases}$ where $0 \leq \theta \leq 2\pi$.

(a) Find the slope of the tangent line at the point where $\theta = \dfrac{\pi}{3}$.

(b) At what points is the tangent line horizontal?

Second Derivative

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \underline{\hspace{6cm}}$$

Example: Compute the second derivative for Γ whenever it's defined and use that to determine concavity of the curve.

Areas

Further suppose that the curve is traced out once by the parametric equations $\begin{cases} x = h(t) \\ y = k(t) \end{cases}$, $\alpha \le t \le \beta$, where $h(t)$ is **monotonic** and $k(t) \ge 0$.

Areas

$$A = \int_a^b y\, dx = \underline{\hspace{8cm}}.$$

Example: Consider the parametric curve (**cycloid**) $\Gamma : \begin{cases} x = 2(\theta - \sin\theta) \\ y = 2(1 - \cos\theta) \end{cases}$ where $0 \le \theta \le 2\pi$. Find the area enclosed by Γ and the $x-$axis.