Idea

Imagine a particle moving along a curve C. It is not always possible to describe a curve by an equation of the form $y = f(x)$. However, the x- and y- coordinates of the particle are functions of time and so we can write $x = f(t)$ and $y = g(t)$. Such a pair of equations is often a convenient way of describing a curve and gives rise to the following definition.

Suppose that x and y are both given as functions of a third variable t (called a **parameter**) by the equations

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$

(called **parametric equations**). Each value of t determines a point (x, y), which we can plot in a coordinate plane. As t varies, the point $(x, y) = (f(t), g(t))$ varies and traces out a curve, which we called a **parametric curve**.

Example:

Sketch and identify the curve defined by the parametric equations $\begin{cases} x = t^2 - 2t \\ y = t + 1 \end{cases}$.

It appears that the curve traced out by the particle may be a ______________________. This can be confirmed by

____.

Visualize with Desmos: https://www.desmos.com/calculator/jx82viodzf

Sometimes we restrict t to lie in a finite interval.

Example:

Sketch and identify the curve defined by the parametric equations $\begin{cases} x = t^2 - 2t \\ y = t + 1 \end{cases}$ where $0 \le t \le 4$.

In general, the curve with parametric equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ where $a \le t \le b$ has **initial point** $\left(f(a), g(a)\right)$ and **terminal point** $\left(f(b), g(b)\right)$.

Example:

What curve is represented by the parametric equations $\begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases}$ where $0 \le \theta \le 2\pi$.

We distinguish between a **curve**, which is a set of points, and a **parametric curve**, in which the points are traced in a particular way.