

Name : _____

Questions 3,4, and 5 are required. Problems 1 and 2 are optional but recommended if you want practice and feedback.

1. Evaluate $\int \frac{1}{x\sqrt{4-x^2}} dx$.

[Solution] $= -\frac{1}{2} \ln \left| \frac{2 + \sqrt{4-x^2}}{x} \right| + C$

2. Evaluate $\int \frac{1}{\sqrt{x^2+16}} dx$. Recommended: figure out your triangle's labels without looking at the table of trig substitution.

[Solution] $= \ln \left| \frac{\sqrt{x^2+16} + x}{4} \right| + C$

3. Evaluate $\int_{\sqrt{2}}^2 \frac{1}{x^3 \sqrt{x^2 - 1}} dx$. Recommended: figure out your triangle's labels without looking at the table of trig substitution.

$$\begin{aligned} \text{[Solution]} &= \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta \right) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} \\ &= \frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4} \end{aligned}$$

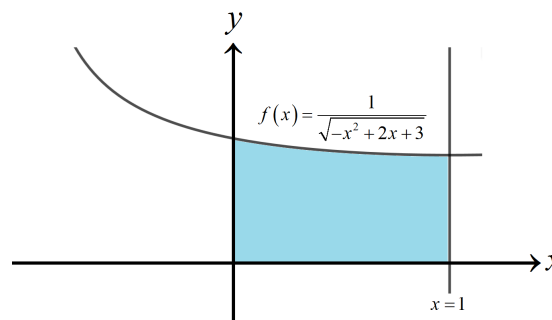
4. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1 + \sin^2 x}} dx$. Recommended: figure out your triangle's labels without looking at the table of trig substitution.

$$\begin{aligned} \text{[Solution]} &= \left(\ln |\sec \theta + \tan \theta| \right) \Big|_0^{\frac{\pi}{4}} \\ &= \ln(\sqrt{2} + 1) \end{aligned}$$

HW due Week 5 Friday, Sept 29

5. Let R be the region bounded by the function $f(x) = \frac{1}{\sqrt{-x^2 + 2x + 3}}$ and x -axis on the interval $[0, 1]$. Find the area of the region R .

Tip: You can directly do inverse trig sub or first do u -substitution with $u = (x-1)/2$



[Solution 1]

$$\begin{aligned} \text{Area} &= \int_0^1 \frac{1}{\sqrt{4 - (x-1)^2}} dx \\ &= \int_{-\frac{\pi}{6}}^0 1 d\theta \\ &= \frac{\pi}{6} \end{aligned}$$

[Solution 2]

$$\begin{aligned} \text{Area} &= \int_0^1 \frac{1}{2\sqrt{1 - \left(\frac{x-1}{2}\right)^2}} dx \\ &= \left(\sin^{-1} u\right)_{-\frac{1}{2}}^0 \\ &= \frac{\pi}{6} \end{aligned}$$