Homework due Week 6 Mon, Oct 2, 2017

Name : ______________________________

1. Evaluate $\int \ln\left(x+\sqrt{1+x^2}\right)dx$.

[Solution]

Hint: Integration by parts. You have no choice but to let $u=\ln\left(x+\sqrt{1+x^2}\right)$ and $dv=dx$.

$$\int \ln\left(x+\sqrt{1+x^2}\right)dx = x\ln\left(x+\sqrt{1+x^2}\right)-\int \frac{x}{\sqrt{1+x^2}}\,dx$$

To solve the right-most integral, do substitution $w=1+x^2$.

Get $\int \ln\left(x+\sqrt{1+x^2}\right)dx = x\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}+C$

2. Evaluate $\int x\tan^2 x\,dx$.

[Solution]

Use trig identity to get $\int x\tan^2 x\,dx = \int x\left(\sec^2 x - 1\right)dx$

$$= \int x\sec^2 x\,dx - \int x\,dx$$

Use integration by parts to get $\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$

$$= x\tan x - \int \frac{\sin x}{\cos x}\,dx$$

Evaluate $\int \frac{\sin x}{\cos x}\,dx$ by substitution (let $w = \cos x$, then $dw = -\sin x\,dx$.)

$$\int x\tan^2 x\,dx = \int x\sec^2 x\,dx - \int x\,dx$$

$$= x\tan x + \ln\left|\cos x\right| - \frac{1}{2}x^2 + C$$

3. Evaluate $\int \cos\sqrt{x}\, dx$

[Solution]

Substitute $w = \sqrt{x}$ and $dw = \dfrac{1}{2\sqrt{x}}\, dx$ to get $\int \cos\sqrt{x}\, dx = 2\int w\cos w\, dw$

Use integration by parts to get $\int w\cos w\, dw = w\sin w - \int \sin w\, dw$

$$= w\sin w + \cos w + c$$

$$\int \cos\sqrt{x}\, dx = 2\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x} + C$$

4. Evaluate $\int x^2 (\ln x)^2 \, dx$.

[Solution]

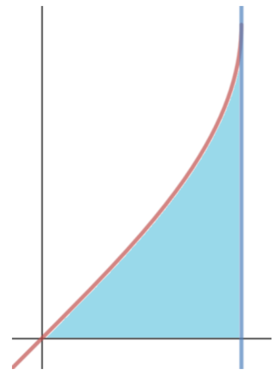
Use integration by parts to get $\int x^2 (\ln x)^2 \, dx = \frac{1}{3}x^3 (\ln x)^2 - \frac{2}{3}\int x^2 \ln x \, dx$

Use integration by parts to get $\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 \, dx$

Therefore,
$$\int x^2 (\ln x)^2 \, dx = \frac{1}{3}x^3 (\ln x)^2 - \frac{2}{3}\int x^2 \ln x \, dx$$
$$= \frac{1}{3}x^3 (\ln x)^2 - \frac{2}{9}x^3 \ln x + \frac{2}{27}x^3 + C$$

5. Consider the graph of the function $f(x) = \sin^{-1} x$. Let R be the region bounded by $y = f(x)$ and x-axis on the interval $[0,1]$.

   Evaluate the **area** of R.



[Solution]

Let $u = \sin^{-1} x$ and $dv = dx$,

then $du = \dfrac{1}{\sqrt{1-x^2}}\, dx$ (verify this is by doing integration by inverse trig substitution).

Therefore, $\int \sin^{-1} x\, dx = x\sin^{-1} x - \int \dfrac{x}{\sqrt{1-x^2}}\, dx$ by above explanation

$= x\sin^{-1} x + \sqrt{1-x^2} + C$ by using substitution $w = 1 - x^2$

The area is $A = \int_0^1 \sin^{-1} x\, dx$

$= \dfrac{\pi}{2} - 1$

6. Evaluate $\int \cos(\ln x)\, dx$. (Hint: the same strategy as Sec 7.1 Ex 4 on pg 474-475.)

[Solution]

Use integration by parts (no choice but to let $u = \cos(\ln x)$ and $dv = dx$),

and get $\int \cos(\ln x)\, dx = x\cos(\ln x) + \int \sin(\ln x)\, dx$

Use integration by parts again to evaluate $\int \sin(\ln x)\, dx$

Combine the two $\int \cos(\ln x)\, dx$. See Sec 7.1 Example 4 on page 474-475.

Get $\int \cos(\ln x)\, dx = \dfrac{x\cos(\ln x) + x\sin(\ln x)}{2} + C$