Name : ______________________________

NOTE 1: Directions/hints/answers are already available on the course website. First attempt without looking at the hints and answers.
NOTE 2: You can type 'series representation of ...' on Wolfram|Alpha.

1. Suppose the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is R. Use the **Test of Your Choice** to find the radius of convergence of the series.
   a. $\sum_{n=1}^{\infty} n c_n x^{n-1}$
   b. $\sum_{n=0}^{\infty} \frac{c_n}{n+1} x^{n+1}$

2. Find a power series representation for the function and determine the interval of convergence. **Pick just ONE of a,b,c, or d. Pick the one that you think will be difficult to do under time pressure (for example, on a test).**
   a. $\frac{5}{1-4x^2}$
   b. $\frac{2}{3-x}$
   c. $\frac{x}{9+x^2}$
   d. $\frac{x}{2x^2+1}$

Do just ONE of e or f.

e. $\dfrac{3}{x^2 - x - 2}$

f. $\dfrac{x+2}{2x^2 - x - 1}$

Do just ONE of g, h, or i. Pick one that you think will be most difficult.

g. $\dfrac{x}{(1+4x)^2}$

h. $\left(\dfrac{x}{2-x}\right)^3$

i. $\dfrac{1+x}{(1-x)^2}$

3. Evaluate the indefinite integral as a power series. What is the radius of convergence?

Do just TWO of the three options.

a. $\int \frac{x}{1+x^3}\,dx$

b. $\int x^2 \ln(1+x)\,dx$

c. $\int \frac{\tan^{-1} x}{x}\,dx$

4. Consider the geometric series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

Do at least (a) and (b).

a. Find the sum of the series $\sum_{n=1}^{\infty} nx^n$ for $|x| < 1$.

b. Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

c. Find the sum of the series $\sum_{n=1}^{\infty} n^2 x^n$ for $|x| < 1$.

d. Find the sum of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

5. It is known that $\cos x$ has a series representation $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$. **Do all parts.**

   a. Find a power series representation for $\cos\sqrt{x}$.

   b. Find a power series representation for $\int \cos\sqrt{x}\, dx$. Hint: Use part (b).

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(continued)

c. Assume that the series you found in part (b) converges for all $x \geq 0$. Use your answer in part (b) to determine a series that represents $\int_0^1 \cos\sqrt{x}\ dx$.

d. If the first **two** non-zero terms of the series are used to estimate the value of the definite integral from part (c), provide a bound on the error of this estimate.

6. It is known that e^x has a series representation $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all real numbers x. **Do all parts.**

   a. Find a power series representation for xe^x.

   b. Find a power series representation for $\frac{d}{dx}\left(xe^x\right)$. Hint: Use part (a).

   c. Evaluate $\sum_{n=0}^{\infty} \frac{(n+1)(-1)^n}{n!}$. Hint: Use part (b).

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d. Find a power series representation for $\int xe^x \, dx$. Hint: Use part (a).

e. Evaluate $\sum_{n=1}^{\infty} \frac{1}{n!(n+2)}$. Hint: Use part (d).