Name : _____

The final answers are given for each problem. Your job is to provide the explanations. You may need extra space – you can attach your own paper.

1. Find the interval and radius of convergence for the power series.

Choose the most challenging series from this page.

a. $\sum_{n=0}^{\infty} \frac{(-2)^n (x+3)^n}{3^{n+1}}$

Answer: Radius of convergence is R = 3/2 and interval of convergence is I = (-9/2, -3/2) both open.

b. $\sum_{n=1}^{\infty} (-1)^n n x^n$

Answer: Radius of convergence is R = 1 and interval of convergence is I = (-1, 1) both open.

c. $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{\sqrt{n}}$

Answer: Radius of convergence is R = 1 and interval of convergence is I = (open) (2, 4] (closed)

Choose the most challenging series from this page.

d. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{7^n (n+1)}$

Answer: Radius of convergence is R = 7 and interval of convergence is I = (open) (-7, 7] (closed)

e. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$

Answer: Radius of convergence is R = 2 and interval of convergence is I = (-2, 2) both open

f. $\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}$

Answer: Radius of convergence is R = 1/10 and interval of convergence is I = [-1/10, 1/10] both closed

Choose the most challenging series from this page.

g. $\sum_{n=0}^{\infty} \frac{n!(x-2)^n}{3^n}$

Answer: Radius of convergence is R = 0 and interval of convergence is I = {2}

h. $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$

Answer: Radius of convergence is R = 4 and interval of convergence is I = open (-4, 4] closed

i. $\sum_{n=1}^{\infty} \frac{n^{20} x^n}{(2n+1)!}$

Answer: interval of convergence is all real numbers

Choose the most challenging power series from this page.

j. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$

Answer: Radius of convergence is R = 1 and interval of convergence is I = closed [1, 3] closed

k. $\sum_{n=1}^{\infty} n!(2x-1)^n$

Answer: Radius of convergence is R = 0 and interval of convergence is I = {1/2}

l. $\sum_{n=1}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$ THERE IS A TYPO. THE STARTING POINT SHOULD BE N=2. OTHERWISE YOU WOULD DIVIDE BY ZERO FOR N=2

Answer: Radius of convergence is R = 1 and the interval of convergence is I = [-1,1] both closed

2. Find the radius of the convergence and the interval of convergence for the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}\left(\frac{x}{2}-1\right)^n$.

Answer: radius of convergence is R = 2 and the interval of convergence is closed [0,4) open

3. The following functions can be represented as power series. Find the interval of convergence for the power series. **Do *both* series.**

   a. $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

Answer: the interval of convergence is (-1,1)

   b. $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$

Answer: all real numbers

Choose a series from this page.

c. $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

Answer: all real numbers

d. $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

Answer: all real numbers

Do *both* series.

e. $\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

Answer: the interval of convergence is [-1,1] both closed.

f. $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

Answer: the interval of convergence is open (-1, 1] closed.