Math 1152Q, Fall 2017 — 11.9 New power series from old

Board work on Week 8, Friday, Oct 20, 2017

Exercise 1. For one of the most-difficult-looking following functions, find a power series representation with center a = 0 by performing one or more operations on a known power series. Also, state the *radius* of convergence of your new power series.

1. $f(x) = \frac{1}{x+2}$ 2. $f(x) = \frac{x^3}{x+2}$ 3. $f(x) = \frac{x}{(1+4x)^2}$

Exercise 2. If the function f has power series representation

$$f(x) = 1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \cdots$$

find a power series representation of

$$g(x) = \frac{f(x) - 1}{x}.$$

Exercise 3. Represent the integral

$$\int_0^{0.5} \frac{1}{1+x^7} \, dx$$

as an infinite series and use the sum of the first four terms as an approximation for the integral. Do you know how good of an approximation this is?