Learning Activity KEY

Name : ________________________________

1. Evaluate $\int \tan^3 x \sec^4 x\, dx$.

[Solution] 1

$$\int \tan^3 x \sec^4 x\, dx = \int \left(\tan^3 x \sec^2 x\right) \sec^2 x\, dx$$
$$= \int \left[\tan^3 x \left(\tan^2 x + 1\right)\right] \sec^2 x\, dx$$

Let $u = \tan x$, then $du = \sec^2 x\, dx$.

Thus $\int \tan^3 x \sec^4 x\, dx = \int \left[\tan^3 x \left(\tan^2 x + 1\right)\right] \sec^2 x\, dx$
$$= \int u^3 \left(u^2 + 1\right) du$$
$$= \int \left(u^5 + u^3\right) du$$
$$= \frac{1}{6} u^6 + \frac{1}{4} u^4 + C$$
$$= \frac{1}{6} \tan^6 x + \frac{1}{4} \tan^4 x + C$$

[Solution] 2

$$\int \tan^3 x \sec^4 x\, dx = \int \left(\tan^2 x \sec^3 x\right) \tan x \sec x\, dx$$
$$= \int \left[\left(\sec^2 x - 1\right) \sec^3 x\right] \tan x \sec x\, dx$$

Let $u = \sec x$, then $du = \tan x \sec x\, dx$.

Thus $\int \tan^3 x \sec^4 x\, dx = \int \left[\left(\sec^2 x - 1\right) \sec^3 x\right] \tan x \sec x\, dx$
$$= \int \left(u^2 - 1\right) u^3\, du$$
$$= \int \left(u^5 - u^3\right) du$$
$$= \frac{1}{6} u^6 - \frac{1}{4} u^4 + C$$
$$= \frac{1}{6} \sec^6 x - \frac{1}{4} \sec^4 x + C$$

2. Evaluate $\int \tan^2 x \sec x \, dx$.

[Solution]

$$\int \tan^2 x \sec x \, dx = \int \left(\sec^2 x - 1\right) \sec x \, dx$$
$$= \int \sec^3 x \, dx - \int \sec x \, dx$$

For $\int \sec^3 x \, dx$, let $u = \sec x$ and $dv = \sec^2 x \, dx$,

then $du = \tan x \sec x \, dx$ and $v = \tan x$.

Thus $\int \tan^2 x \sec x \, dx = \int \sec^3 x \, dx - \int \sec x \, dx$

$$= \left(\tan x \sec x - \int \tan^2 x \sec x \, dx \right) - \int \sec x \, dx$$
$$= \tan x \sec x - \int \tan^2 x \sec x \, dx - \int \sec x \, dx$$

As a result, $2\int \tan^2 x \sec x \, dx = \tan x \sec x - \int \sec x \, dx$

Therefore, $\int \tan^2 x \sec x \, dx = \frac{1}{2} \tan x \sec x - \frac{1}{2} \int \sec x \, dx$

$$= \frac{1}{2} \tan x \sec x - \frac{1}{2} \ln\left|\tan x + \sec x\right| + C$$