Learning Activity KEY

Name : ______________________________

1. Evaluate $\int \sin^3 x \cos^5 x \, dx$.

[Solution] 1

$$\int \sin^3 x \cos^5 x \, dx = \int \left(\sin^3 x \cos^4 x\right) \cos x \, dx$$
$$= \int \left[\sin^3 x \left(\cos^2 x\right)^2\right] \cos x \, dx$$
$$= \int \left[\sin^3 x \left(1-\sin^2 x\right)^2\right] \cos x \, dx$$

Let $u = \sin x$, then $du = \cos x \, dx$.

Thus $\int \sin^3 x \cos^5 x \, dx = \int \left[\sin^3 x \left(1-\sin^2 x\right)^2\right] \cos x \, dx$

$$= \int u^3 \left(1-u^2\right)^2 \, du$$
$$= \int u^3 \left(1-2u^2+u^4\right) du$$
$$= \int \left(u^3 - 2u^5 + u^7\right) du$$
$$= \frac{1}{4}u^4 - \frac{1}{3}u^6 + \frac{1}{8}u^8 + C$$
$$= \frac{1}{4}\sin^4 x - \frac{1}{3}\sin^6 x + \frac{1}{8}\sin^8 x + C$$

[Solution] 2

$$\int \sin^3 x \cos^5 x \, dx = \int \left(\sin^2 x \cos^5 x\right) \sin x \, dx$$
$$= \int \left[\left(1-\cos^2 x\right)\cos^5 x\right] \sin x \, dx$$

Let $u = \cos x$, then $du = -\sin x \, dx$.

Thus $\int \sin^3 x \cos^5 x \, dx = \int \left[\left(1-\cos^2 x\right)\cos^5 x\right] \sin x \, dx$

$$= -\int \left(1-u^2\right) u^5 \, du$$
$$= -\int \left(u^5 - u^7\right) du$$
$$= -\frac{1}{6}u^6 + \frac{1}{8}u^8 + C$$
$$= -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + C$$

2. Evaluate $\int \sin^2 x \cos^2 x \, dx$.

[Solution]

$$\begin{aligned}\int \sin^2 x \cos^2 x \, dx &= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx \\ &= \frac{1}{4}\int \left(1-\cos^2 2x\right) dx \\ &= \frac{1}{4}\int \sin^2 2x \, dx \\ &= \frac{1}{4}\int \frac{1-\cos 4x}{2} \, dx \\ &= \frac{1}{8}\left(x - \frac{1}{4}\sin 4x\right) + C \\ &= \frac{1}{8}x - \frac{1}{32}\sin 4x + C\end{aligned}$$