Learning Activity

1. Consider the geometric series
$$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n \text{ for } |x|<1$$
   a. Find a power series representation for $\frac{1}{1+x^2}$ and give the *interval* of convergence (from Week 8 Mon reading homework).
   b. Find a power series representation for $\tan^{-1} x$ and give the *interval* of convergence. (Hint: use term-by-term differentiation or integration).
   c. Evaluate the infinite series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$. Hint: use part (b).

Solution:

a. Use Theorem (composition), follow Example 1, pg 753.
b. Use Theorem (term-by-term integration), follow Example 7, pg 756 to determine the radius of convergence, R=1. To determine the interval of convergence, check the endpoints x=-1 and x =1. Using the Alternating Series Test for both endpoints, you get that the series converges when x=-1 and also when x=1. Hence the interval of convergence is [-1,1].
c. You answer to part (b) is a power series which converges to the function arctan(x) for x in the interval [-1,1]. To answer to part (c), simply "plug in" x=1 into the series from part (b) which is equal arctan(1) = pi/4.