(WebAssign 9.1 differential equations)

1. For what values of k does the function $y = \cos(kt)$ satisfy the differential equation $4y'' = -9y$?

2. Circle all functions which are solutions to $4y'' = -9y$. (If none or all, state so).

   (a) $y = -\cos(\frac{3t}{2})$
   Answer: Yes

   (b) $y = \cos(\frac{3t}{2}) + 1$
   Answer: No

   (c) $y = \sin(\frac{3t}{2})$

   (d) $y = \sin(\frac{3t}{2}) + \cos(\frac{3t}{2})$

3. True or false? Every member of the family of functions $y = \dfrac{4\ln(x) + C}{x}$ is a solution of the differential equation

$$x^2 y' + xy = 4$$

   Show work.

4. Find a solution of the differential equation that satisfies the initial condition $y(1) = 2$. Answer $y = \frac{4\ln(x)+2}{x}$.

5. Find a solution of the differential equation that satisfies the initial condition $y(2) = 1$.

6. Find a solution of the differential equation that satisfies the initial condition $y(3) = 1$.

7. What can you say about a solution of the differential equation $y' = -\frac{1}{2}y^2$ *just* by looking at the differential equation? Circle all possibilities.

   (a) The function y must be equal to 0 on any interval on which it is defined.
   Answer: no.

   (b) The function y must be strictly increasing on any interval on which it is defined.

   (c) The function y must be increasing (or equal to 0) on any interval on which it is defined.

   (d) The function y must be decreasing (or equal to 0) on any interval on which it is defined.

   (e) The function y must be strictly decreasing on any interval on which it is defined.

8. Verify that all members of the family $y = \frac{2}{x+C}$ are solutions of the differential equation $y' = -\frac{1}{2}y^2$.

9. Write a solution of the differential equation $y' = -\frac{1}{2}y^2$ that is not a member of the family $y = \frac{2}{x+C}$. Hint: See Problem 2a and 3a on In-class 9.1 Worksheet `https://egunawan.github.io/fall17/notes/1132q9_1_modeling_with_differential_equations.pdf`

10. Find a solution of the initial-value problem. $y' = -\frac{1}{2}y^2 \qquad y(0) = 0.1$
Answer: $\frac{2}{x+20}$

11. Find a solution of the initial-value problem. $y' = -\frac{1}{4}y^2 \qquad y(0) = 0.2.$
Hint: the solution will be in the form of $\frac{4}{x+K}$

12. Find a solution of the initial-value problem. $y' = -\frac{1}{3}y^2 \qquad y(0) = 0.5$
Hint: the solution will be in the form of $\frac{3}{x+K}$

13. Find a solution of the initial-value problem. $y' = -\frac{1}{6}y^2 \qquad y(0) = 0.5$
Hint: the solution will be in the form of $\frac{6}{x+K}$

14. A population is modeled by the differential equation

$$\frac{dP}{dt} = 1.1P\left(1 - \frac{P}{4000}\right)$$

(a) For what values of P is the population increasing?
Answer: $(0, 4000)$. Explanation: You need $1 - P/4000 > 0$ and $P > 0$.

(b) For what values of P is the population decreasing?

(c) What are the equilibrium solutions? Hint: You need $dP/dt = 0$.

15. A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y^4 - 8y^3 + 15y^2.$$

(a) What are the constant solutions of the equation?
Answer: $y = 0, y = 3, y = 5$

(b) Sketch the polynomial $t^4 - 8t^3 + 15t^2$. In particular, mark the x-intercepts.

(c) For what values of y is y increasing?

(d) For what values of y is y decreasing?
Answer: $(3, 5)$