7.2 Trigonometric Integrals

Trigonometric Identities and Formulas.

1. $\sin^2\theta + \cos^2\theta = 1$	3. $\sin 2\theta = 2\sin\theta\cos\theta$	5. $\cos^2\theta = \frac{1+\cos 2\theta}{2}$
2. $\tan^2\theta + 1 = \sec^2\theta$	4. $\cos 2\theta = \cos^2\theta - \sin^2\theta$	6. $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

1. **Example:** Evaluate $\displaystyle\int_0^{\pi} \sin^3(5x)\,dx$.

   Thinking about the problem:

   What technique of integration should I use to evaluate the integral and why? Have I seen a problem similar to this one before? If so, which technique did I use?

   I know that I need to evaluate a definite integral so my result should result in a number. To determine which technique to use, I will focus on the integrand, i.e., $\sin^3(5x)$. In this case, I cannot directly integrate $\displaystyle\int_0^{\pi} \sin^3(5x)\,dx$, so I will consider the Trigonometric Identities. I will need to use an identity which involves powers of $\sin(x)$. I think I will use the identity $\sin^2\theta + \cos^2\theta = 1$, so $\sin^2\theta = 1 - \cos^2\theta$. Then I can find $\sin^3(5x) = \sin(5x)\sin^2(5x) = \sin(5x)(1-\cos^2(5x))$. Since $\sin^3(5x) = \sin(5x)(1-\cos^2(5x))$, I find that $\displaystyle\int_0^{\pi} \sin^3(5x)\,dx = \int_0^{\pi} \sin(5x)(1-\cos^2(5x))\,dx$. If this new integral is more difficult to solve than my original integral, then I probably made a mistake in my choice of the identity to use. However, I see that I can integrate $\displaystyle\int_0^{\pi} \sin(5x)(1-\cos^2(5x))\,dx$ using the technique of substitution, which is how I will proceed to evaluate the integral.

Doing the problem:

The problem asks to evaluate the integral. I note that I can use the identity $\sin^2\theta + \cos^2\theta = 1$ and find

$$\int_0^\pi \sin^3(5x)\,dx = \int_0^\pi \sin(5x)(1-\cos^2(5x))dx.$$

Then I use the substitution $u = \cos(5x)$ so that $du = -5\sin(5x)dx$. I also evaluate the bounds $\cos(5\cdot 0) = 1$ and $\cos(5\cdot\pi) = -1$ to find the new integral

$$\begin{aligned}
\int_0^\pi \sin(5x)(1-\cos^2(5x))dx &= \int_1^{-1} \sin(5x)(1-u^2)\frac{du}{-5\sin(5x)} \\
&= -\frac{1}{5}\int_1^{-1} 1-u^2\,du \\
&= \frac{1}{5}\int_{-1}^{1} 1-u^2\,du \\
&= \frac{1}{5}\left[u - \frac{u^3}{3}\right]_{-1}^{1} \\
&= \frac{1}{5}\left(\left[1-\frac{1}{3}\right] - \left[-1-\frac{-1}{3}\right]\right) \\
&= \frac{1}{5}\left(2-\frac{2}{3}\right) \\
&= \frac{4}{15}.
\end{aligned}$$

Solutions should show all of your work, not just a single final answer.

2. Identify the trig identities to simplify the following integrands:

   (a) $\int \sin^2 x \, dx$

   (b) $\int \cos^2 x \, dx$

3. Evaluate $\int \sin^2 x \cos^2 x \, dx$. (*Hint:* Look at the exponents. Are they the same?)

   (a) Which identity would you use to simplify the integrand?

   (b) Using part (a), simplify the integrand.

   (c) Evaluate $\int \sin^2 x \cos^2 x \, dx$ using the integrand found in part (b).

4. Evaluate $\int_0^{\pi} \cos^3 x \, dx$.

5. Evaluate $\int \cos x \sin^2 x \, dx$. (*Hint:* There may be more than one technique to evaluate this integral.)

   (a) State the techniques you could use to evaluate the integral.

   (b) Choose the most efficient technique and evaluate $\int \cos x \sin^2 x \, dx$.

6. T/F (with justification): The value of $\int_{-\pi}^{\pi} \sin^9 x \, dx$ is 0.