10.1 Curves Defined by Parametric Equations

1. **Example:** In the following parametric equations, eliminate the parameter to obtain a single equation only in terms of x and y. Then describe the graph it traces.

$$x = -\sin(2t), \quad y = \cos(2t); \quad 0 \leq t \leq 2\pi.$$

Thinking about the problem:

How should I eliminate the parameter t? Have I seen a problem similar to this one before? If so, what approach did I use?

I notice that since x and y look like parametric equations of a circle, I should expect the single equation in terms of x and y to look like a circle. To describe the graph, I will see my resulting equation in terms of x and y to find the graph it traces, but use the individual x and y to find the orientation and starting point of my parametrization.

Doing the problem:

I know that $\sin^2(2t)+\cos^2(2t) = (-\sin(2t))^2+(\cos(2t))^2 = 1$, so replacing $x = -\sin(2t)$ and $y = \cos(2t)$, I find the resulting equation $x^2 + y^2 = 1$ and the equation is only in terms of x and y. So I know my graph will be a circle of radius 1 centered at the origin. I know that since $x = -\sin(2 \cdot 0) = 0$ and $y = \cos(2 \cdot 0) = 1$, the starting point of my parametrization is $(0, 1)$. Similarly, I know that $x = -\sin(2t)$ starts out negative whereas $y = \cos(2t)$ starts positive, so my orientation will be x starting at 0 and going negative (left) and y starting at 1 and staying positive. Therefore my parametrization is counterclockwise. Finally, since $0 \leq t \leq 2\pi$, I see that my parametrization traces the circle twice, since $x = -\sin(2t)$ and $y = \cos(2t)$ indicates my parametrization goes twice the speed as $\sin(t)$ and $\cos(t)$, which would trace the graph once in $0 \leq t \leq 2\pi$.

Solutions should show all of your work, not just a single final answer.

2. In the following parametric equations, eliminate the parameter to obtain a single equation only in terms of x and y.

$$x = t^2 + 4, \quad y = 3t^2; \quad 0 \leq t \leq 2.$$

   (a) Write t in terms of x.

   (b) Write t in terms of y.

   (c) Set your answer to (a) and (b) equal to each other.

3. In the following parametric equations, eliminate the parameter to obtain a single equation only in terms of x and y.

$$x = 3\cos t, \quad y = 3\sin t; \quad 0 \leq t \leq \frac{\pi}{2}.$$

4. Find a parameterization of the following: a circle centered at the origin with radius 16, oriented counterclockwise with initial point $(0, 16)$.

   (a) What are the parametric equations for a circle?

   (b) Alter the parametric equations from (a) to trace a circle with radius 16.

   (c) Alter the parametric equations from (b) to be oriented counterclockwise.

   (d) Alter the parametric equation from (c) so when $t = 0$, $x = 0$ and $y = 16$.

5. Find a parameterization of the following: a circle centered at $(-2,-3)$ with radius 8, oriented clockwise with initial point $(6,-3)$.

6. T/F (with justification)

   The parametric curve $(\sin t, -\cos t)$ as t increases traces out a circle counterclockwise.