Math 1152Q | Due Week 8 Friday, Oct 13 | Problems B | Fall 2017

You may use your own paper so that you can have more space. You are encouraged to discuss your work with your classmates but this assignment is to be completed individually (that is, don't share/ let someone take a picture of your full solutions).

1. (Credit)

   Please give credit to people that you talked with (including the people that *you* helped and the people who gave you help), any textbooks you used (other than Stewart), and the website addresses of any internet resources you might have used (other than the pages linked to this file).

2. (Style points) You will earn full "Style points" if each of your solution is legible, coherent, and not ambiguous. In addition, your reader should not need to reread your solution several times to find a train of thought. One tip is that your final draft should not include any scratch work.

3. Suppose you have a large supply of rectangles (like books or a deck of cards), all the same size, and you stack them at the edge of a table, with each book extending farther beyond the edge of the table than the one beneath it. Our goal is to show that it is possible to do this so that the top book or card can extend any distance at all beyond the edge of the table if the stack is high enough.

   Use the following method of stacking: the top book extends half its length beyond the second book. The second book extends a quarter of its length beyond the third. The third extends one-sixth of its length beyond the fourth, and so on. See `https://d3njjcbhbojbot.cloudfront.net/api/utilities/v1/imageproxy/https://coursera-course-photos.s3.amazonaws.com/37/d1a6e9c4c9f32a3826ceaeb94dd79e/self-describing-sequence-and-harmonic-series.png`.

   a.) (bonus) For this activity, you are encouraged to get together with a classmate or two (but no more than three people total including yourself). Try this activity yourself with a deck of cards so that the top card extends beyond the bottom card. To receive the bonus points, take a picture of your work. All the participants (no more than 3) should be in the picture, and the picture should show clearly that the top object extend beyond the end of your bottom object.

   b.) Afterwards, watch this Coursera video: "how far can you build a one-sided bridge?" `https://www.coursera.org/learn/advanced-calculus/lecture/9AUVI/how-far-out-can-you-build-a-` (You don't need to sign up to view the video - just click exit whenever a pop-up appears - and you can watch at faster speed by clicking the setting button on the bottom right of the page).

Your job: Write a letter (one page or less) to a friend (who is also taking Calc II) the gist of what the video is about. You must draw pictures (or insert pictures from elsewhere if you don't like to draw).

4. Dialysis treatment removes waste products from a patient's blood by diverting some of the bloodflow externally through a machine called a dialyzer. The rate at which urea is removed from the blood (in mg/minute) is often well described by the equation

$$u(t) = \frac{r}{V}\, C_0\, e^{(-rt/V)} \quad \text{where}$$

r is the constant rate of flow of blood through the dialyzer (in mL/minute), V is the volume of the patient's blood (in mL), and C_0 is the amount of urea in the blood (in mg) at time $t = 0$.

a.) Evaluate the improper integral

$$\int_0^\infty u(t)\ \mathrm{dt}.$$

b.) Interpret your answer (in relation to this removal process). Make sure the interpretation makes sense (in particular, the amount of waste that can be removed is limited).

5. Find a positive number N so that, if $n > N$, then we have

$$\int_n^\infty \frac{1}{x^2+1}\,\mathrm{dx} < 0.001.$$

(Note: N does not need to be an integer - any positive real number that satisfies the inequality above would work! You don't need to specify the number N using digits).

6. **Definition**: Let $f(x)$ be continuous on the interval $[0, \infty)$. The *Laplace transform of f* is defined to be a function $F : D \to \{$ real number $\}$ where

$$F(s) = \int_0^\infty f(t)\ e^{-st}\ \mathrm{dt},$$

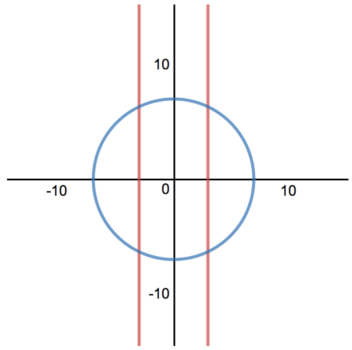
and the domain of F is $D = \{$all numbers s for which the improper integral converges$\}$.
Your task: for each function below, compute its Laplace transform.

a.) $f(t) = 1$

b.) $g(t) = e^t$

c.) $h(t) = t$

7. Suppose you and your two classmates have a circular cookie (with a 14-cm diameter). You want to divide it equally among the three of you using only vertical slices $x = a$ and $x = -a$. Where should you make the slices? That is, what should a be?



a.) Set up the equation (after evaluating a definite integral) so that you can compute an approximation with a computing tool. You do not need a computer for this part. (Recall how to represent a circular shape: https://www.desmos.com/calculator/ilx2h6ilkv)

b.) Use a graphing tool to approximate where you should make the slices. Explain in words how you came up with the number. Include a printout of your graph or sketch it.

8. For $n = 1, 2, 3, \ldots$, let a_n be defined by the formula

$$a_n \overset{\text{def}}{:=} \lim_{t\to\infty} \left(\int_{\frac{1}{t}}^{1} (\ln x)^n \, \mathrm{dx} \right).$$

Note that the right-hand-side is just the definition of the improper integral $\int_0^1 (\ln x)^n \, \mathrm{dx}$. Your goal is to (informally) explain that

$$a_n \overset{\text{needs to show}}{=} (-1)^n \, n! \quad \text{for all } n \geq 1$$

You'll need to show step-by-step computations using integration by parts and limit laws.

a.) First, show that $a_n \overset{\text{show}}{=} (-1)^n \, n!$ holds for just $n = 1$ or $n = 2$. That is, show that $a_1 \overset{\text{show}}{=} -1$ or show that $a_2 \overset{\text{show}}{=} 2$.

b.) Next, show that IF k is a positive integer and $a_k = (-1)^k \, k!$, THEN $a_{k+1} = (-1)^{k+1} \, (k+1)!$

Step 1: Suppose that k is a positive number. **Assume** that $a_k \overset{\text{given}}{=} (-1)^k \, k!$. Warning: You do not get to pick a particular number for k.

Step 2: Use the given equation in Step 1 to show that $a_{k+1} \overset{\text{show}}{=} (-1)^{k+1} \, (k+1)!$ using familiar integration techniques.

(Note: This would take a lot of space, so feel free to use your own piece of paper)

(Hint: As a warm-up, practice step 2 on your notebook with $k = 4$. Given that $a_4 \overset{\text{given}}{=} (-1)^4 \, 4! = 4!$, show that $a_5 \overset{\text{show}}{=} (-1)^5 \, 5!$ USING the given information that $a_4 \overset{\text{given}}{=} (-1)^4 \, 4! = 4!$ and L'Hospital's Rule).

(extra space)

9. (Write your own problem) Review the topics of chapter 7.

   a.) Write a (non-vocabulary) problem that is more difficult than a reading homework question. It should require a decent understanding of one or more of the concepts/ skills to solve. (For ideas, you may look at Stewart's textbook problems or other textbooks or WebAssign).

   b.) Explain briefly why you chose to highlight certain concepts (for example, maybe it's a brand-new concept to you or has applications to science).

   c.) Write a complete solution to your problem.

   d.) Show your problem to your classmates to get their feedback. Write a brief, one-sentence comment about what other people thought. One or two questions submitted from students may be chosen for future test questions.