Name : ____________________________

(Graded on **correctness** - you can use the internet and share solutions as long as it leads you to learn or re-learn these basic topics - you do not have to show work except on question 11).

If you can't find it in the book and Googling doesn't help, please ask me!

1. The exact value of $\left(\frac{1}{2}+\frac{1}{3}\right) \div \frac{5}{4}$ is ______________ . **Answer**: 2/3

2. (motivation: partial fraction decomposition)

   Solve the system of linear equations $\begin{cases} 2A+B+C &= 1 \\ A+2B+C &= 3 \\ A+B+2C &= 4 \end{cases}$.

   Answer: $A=-1, B=1, C=2$

3. (motivation: integral by trig substitution when you need to complete the square)
   Consider the parabola $y = x^2 - 3x + 5$.

   The vertex of the parabola is located at ______________ on the $xy-$plane.

   Answer: $(\frac{3}{2}, \frac{11}{4})$

4. (motivation: integral by trig substitution when you need to use trig identities)
   Which of the following equals $1 - \dfrac{1}{\tan^2 x + 1}$?

   A. $\sin^2 x$ B. $\cos^2 x$

   C. $\tan^2 x$ D. $\sec^2 x$

   (Hint: the back of the book will give you the trig identity $1 + \tan^2\theta = \sec^2\theta$).
   Answer: $\sin^2 x$

5. (motivation: polar coordinate, finding area)

   The area of a sector with central angle 30° ($\frac{\pi}{6}$) in a circle of radius 12 m is ________ m^2.

   Answer: $12\pi m^2$

6. For $-1 \le x \le 1$, the algebraic expression (without trig expressions) of $\sin\left(2\cos^{-1} x\right)$ is ...
   (Hint: use the identity $\sin^2(\theta) = 1 - \cos^2(\theta)$, the definition $\cos(\cos^{-1} x) = x$,
   and the double angle formula $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.
   Motivation: See trig substitution method Sec 7.3. Example 1 pg 486-487).

Answer (option 1):

$$\begin{aligned}\sin(2\cos^{-1}x) &= \sqrt{1-\cos^2(2\cos^{-1}x)} \text{ use } \sin^2(\theta)=1-\cos^2(\theta), \text{ or draw a } \triangle \text{ with angle } \cos^{-1}(x)\\ &= \sqrt{1-[\cos(2\cos^{-1}x)]^2}\\ &= \sqrt{1-(2\cos^2(\cos^{-1}x)-1)^2} \text{ using } 2\cos^2(\theta)-1=\cos 2\theta\\ &= \sqrt{1-(2x^2-1)^2}\\ &= \sqrt{1-(4x^4-4x^2+1)}\\ &= \sqrt{-4x^4+4x^2)}\\ &= \sqrt{4x^2(-x^2+1)}\\ &= 2x\sqrt{-x^2+1}\end{aligned}$$

Answer (option 2):

$$\begin{aligned}\sin(2\cos^{-1}x) &= 2\sin(\cos^{-1}x)\cos(\cos^{-1}x) \text{ using } \sin 2\theta = 2\sin\theta\cos\theta\\ &= 2\sin(\cos^{-1}x)x\\ &= 2x\left(\sqrt{1-\cos^2(\cos^{-1}x)}\right) \text{ draw } \triangle \text{ with angle } \cos^{-1}(x), \text{ or use } \sin^2 x+\cos^{-1}x=1\\ &= 2x(\sqrt{1-x^2})\end{aligned}$$

If you think of other ways to do this, let me know and I'll put it here:

7. (motivation: integrating square of sine/cosine using half-angle formulas)
Find the exact values of the following. You can look up half-angle formulas in the back of the book or ask Google.

(a) $\sin^2\frac{\pi}{8} =$ ______________.
Answer: $\frac{2-\sqrt{2}}{4}$

(b) $\cos^2\frac{\pi}{8} =$ ______________.
Answer: $\frac{2+\sqrt{2}}{4}$

8. (motivation: polar coordinate when you need to convert Cartesian coordinate to polar coordinate)
Write the complex number $z = 1 - i$ (that is, the point where $x = 1$ and $y = -1$ on the Cartesian plane) in polar form with argument θ between 0 and 2π :
Answer: $\sqrt{2}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$ (I will explain this later in Chapter 8)

9. (motivation: reviewing derivative of exp(x), ln(x), sin(x), cos(x), constant, and chain rule, product rule, power rule)
The second derivative of the function $f(x) = e^{\sin x} + \ln x + \pi^2$ is
Answer: $f''(x) = e^{\sin x}cos^2x - e^{\sin x}\sin x - \frac{1}{x^2}$

10. (motivation: when they want to show that a given sequence is decreasing)
Find the interval(s) on which the function $f(x) = 4x^3 - 15x^2 - 72x + 5$ is decreasing :
Answer: $(-\frac{3}{2}, 4)$

11. Review u-substitution by doing one of these (or both):
    - watch Khan Academy video `https://www.khanacademy.org/math/calculus-home/ap-calculus-ab/ab-antiderivatives-ftc#ab-u-substitution`
    - go to Sec 5.5 pages 413-416 and test yourself on Examples 1-6.

    Pick two examples from pages 413-416 or the Khan academy video and redo them below without looking at the book/ your note/ video:

12. Evaluate the following.
    (a) $\lim_{x\to\infty} \frac{x^2 + x - 2}{x^2 - 1} =$ ____ (compare the coefficients of the highest power of x).
    Answer: 1
    (b) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} =$ ____________ (practice L'Hospital's Rule).
    Answer: 0
    (c) $\int_0^1 \frac{1}{1+x^2}\, dx =$ ____________
    (A very common integral. Look this up on the back of the book or ask WolframAlpha).
    Answer: $\frac{\pi}{4}$
    (d) $\int_0^5 x\sqrt{25 - x^2}\, dx =$ ____________ (review u-substitution).
    Answer: $\frac{125}{3}$

13. (motivation: used in all the integration skills. It's a good habit to check whether your answer is correct or wrong.)
Tyrion Lannister evaluated the following indefinite integral during the Battle of Calculus :

$$\int x \cos x\, dx = x \sin x + \cos x + C.$$

You don't know how he came up with this result, but you do know whether his answer is correct or not. Why?

Answer:

$$\begin{aligned} \frac{d}{dx}(x \sin x + \cos x + C) &= \sin x + x \cos x - \sin x + 0 \\ &= x \cos x. \end{aligned}$$