

1. (9.3 from reading homework)

- (a) Draw a rough sketch of a possible solution to the logistic differential equation  $\frac{dP}{dt} = 5P \left(1 - \frac{P}{8}\right)$ .

**You do not need to solve this differential equation to draw a rough sketch.** Hint:

Explained in <https://www.khanacademy.org/math/ap-calculus-bc/bc-diff-equations/bc-logistic-models/e/logistic-differential-equation>

2. (9.3 from WebAssign)

- (a) Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dx} = \frac{x}{y}, \quad y(0) = -9$$

Answer:  $y = -\sqrt{x^2 + 81}$

- (b) Find the solution of the differential equation that satisfies the given initial condition.

$$xy' + y = y^2, \quad y(1) = -8$$

Answer:  $y = \frac{8}{8-9x}$

- (c) Consider the differential equation

$$(x^2 + 15)y' = xy$$

1. Find all constant solutions.

Answer:  $y = 0$

2. Find all solutions.

Answer:  $y = K\sqrt{x^2 + 15}$

- (d) The differential equation below models the temperature of a  $86^\circ$  C cup of coffee in a  $20^\circ$  C room, where it is known that the coffee cools at a rate of  $1^\circ$  C per minute when its temperature is  $70^\circ$  C. Solve the differential equation to find an expression for the temperature of the coffee at time  $t$ . (Let  $y$  be the temperature of the cup of coffee in  $^\circ$ C, and let  $t$  be the time in minutes, with  $t = 0$  corresponding to the time when the temperature was  $86^\circ$  C.)

$$\frac{dy}{dt} = -\frac{1}{50}(y - 20)$$

Answer:  $y = Ke^{-t/50} + 20$ . After considering the initial condition, we see that the temperature of the coffee at the time is described by  $y = 66e^{-t/50} + 20$ .

- (e) A tank contains 8000 L of brine with 14 kg of dissolved salt. Pure water enters the tank at a rate of 80 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate.

1. How much salt is in the tank after  $t$  minutes?

Answer:  $y = 14e^{-t/100}$  kg

2. How much salt is in the tank after 20 minutes?

Answer:  $14e^{-0.2}$  kg. (Around 11.5 kg). You don't need to approximate.

- (f) 1. Find the orthogonal trajectories of the family of curves

$$y^2 = 8kx^3$$

Answer:  $2x^2 + 3y^2 = C$

2. Sketch these orthogonal trajectories.