

1. (9.3 from reading homework)

- (a) Draw a rough sketch of a possible solution to the logistic differential equation  $\frac{dP}{dt} = 5P\left(1 - \frac{P}{8}\right)$ . **You do not need to solve this differential equation to draw a rough sketch.**

Hint: Explained in <https://www.khanacademy.org/math/ap-calculus-bc/bc-diff-equations/bc-logistic-models/e/logistic-differential-equation>

2. (9.3 from WebAssign)

- (a) Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dx} = \frac{x}{y}, \quad y(0) = -9$$

- (b) Find the solution of the differential equation that satisfies the given initial condition.

$$xy' + y = y^2, \quad y(1) = -8$$

- (c) Consider the differential equation

$$(x^2 + 15)y' = xy$$

1. Find all constant solutions.
  2. Find all solutions.
- (d) The differential equation below models the temperature of a  $86^\circ$  C cup of coffee in a  $20^\circ$  C room, where it is known that the coffee cools at a rate of  $1^\circ$  C per minute when its temperature is  $70^\circ$  C. Solve the differential equation to find an expression for the temperature of the coffee at time  $t$ . (Let  $y$  be the temperature of the cup of coffee in  $^\circ$ C, and let  $t$  be the time in minutes, with  $t = 0$  corresponding to the time when the temperature was  $86^\circ$  C.)

$$\frac{dy}{dt} = -\frac{1}{50}(y - 20)$$

- (e) A tank contains 8000 L of brine with 14 kg of dissolved salt. Pure water enters the tank at a rate of 80 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate.

1. How much salt is in the tank after  $t$  minutes?
2. How much salt is in the tank after 20 minutes?

- (f) 1. Find the orthogonal trajectories of the family of curves

$$y^2 = 8kx^3$$

2. Sketch these orthogonal trajectories.