1. True or False.

   (a) If $-1 < \alpha < 1$, then $\lim_{n\to\infty} \alpha^n = 0$. **T** **F**
   Justification:

   (b) If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges. **T** **F**
   Justification:

   (c) if $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent. **T** **F**
   Justification:

   (d) if $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n b_n\}$ is divergent. **T** **F**
   Justification:

   (e) If $a_n > 0$ and $\{a_n\}_{n=1}^{\infty}$ is decreasing, then $\sum a_n$ converges **T** **F**
   Justification:

   (f) The series $\sum_{n=1}^{\infty} n^{-\sin 1}$ converges **T** **F**
   Justification:

   (g) The series $\sum_{n=1}^{\infty} n^{-\sin 1}$ diverges. **T** **F**
   Justification:

   (h) The series $\sum_{n=1}^{\infty} n^{-\cos 0}$ diverges. **T** **F**
   Justification:

   (i) If $a_n > 0$ and $\sum a_n$ converges, then $\sum (-1)^n a_n$ converges **T** **F**
   Justification:

   (j) The ratio test can be used to determine whether $\sum \frac{1}{n^4}$ converges. **T** **F**
   Justification:

   (k) The ratio test can be used to determine whether $\sum \frac{1}{n!}$ converges. **T** **F**
   Justification:

   (l) If $a_n > 0$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then $\lim_{n\to\infty} a_n = 0$ **T** **F**
   Justification:

   (m) If $a_n > 0$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0$, then $\lim_{n\to\infty} a_n = 0$ **T** **F**
   Justification:

   (n) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent. **T** **F**
   Justification:

   (o) If $a_n > 0$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then $\lim_{n\to\infty} a_n = 0$ **T** **F**
   Justification:

   (p) $0.99999\cdots = 1$ **T** **F**
   Justification:

   (q) $0.9999\cdots \neq 1$ **T** **F**
   Justification:

   (r) 0.66 is close to $\frac{2}{3}$ but $0.66 \neq \frac{2}{3}$ **T** **F**
   Justification:

   (s) $0.66666\ldots$ is close to $\frac{2}{3}$ but $0.66666\cdots \neq \frac{2}{3}$ **T** **F**
   Justification:

   (t) If 200 terms are added to a convergent series, the new series is still convergent **T** **F**
   Justification:

   (u) If 200 terms are removed from a divergent series, the new series is covergent **T** **F**
   Justification:

2. (7.8 improper integral)

   i. When is an integral improper?

ii. One or more of the integrals below is/are improper. Circle all the *improper* integrals and evaluate.

$$\int_2^3 \sqrt{x-2}\,\text{dx}, \quad \int_0^1 \frac{27}{x^5}\,\text{dx}, \quad \int_{-1}^1 \frac{1}{x}\,\text{dx}, \quad \int_2^\infty \left(\frac{1}{e^5}\right)^x \text{dx}, \quad \int_2^\infty \frac{1}{x^2+8x-9}\,\text{dx}$$

$$\int_0^1 \frac{4}{x^5}\,\text{dx}, \quad \int_0^1 \frac{4}{x^{0.5}}\,\text{dx}, \quad \int_2^3 \frac{2}{\sqrt{3-x}}\,\text{dx}, \quad \int_4^8 \frac{4}{x\sqrt{x^2-16}}\,\text{dx}, \quad \int_{-7}^7 \frac{1}{\sqrt{49-x^2}}\,\text{dx}.$$

iii. Write 2 improper (definite) integrals (different from above) so that one is convergent and the other is divergent.

iv. Write 2 proper (definite) integrals that are different from above.

v. Write 2 indefinite integrals.

vi. Determine whether $\int_0^1 9x^2 \ln(x)\,\text{dx}$ converges or diverges. If it converges, evaluate it.

3. (WebAssign 9.1 differential equations)

(a) For what values of k does the function $y = \cos(kt)$ satisfy the differential equation $4y'' = -9y$?

(b) Circle all functions which are solutions to $4y'' = -9y$. (Possibly none or all).

1. $y = -\cos(\frac{3t}{2})$
2. $y = \cos(\frac{3t}{2}) + 1$
3. $y = \sin(\frac{3t}{2})$
4. $y = \sin(\frac{3t}{2}) + \cos(\frac{3t}{2})$

(c) True or false? Every member of the family of functions $y = \dfrac{4\ln(x) + C}{x}$ is a solution of the differential equation

$$x^2 y' + xy = 4$$

(d) Find a solution of the differential equation that satisfies the initial condition $y(1) = 2$.

(e) Find a solution of the differential equation that satisfies the initial condition $y(2) = 1$.

(f) Find a solution of the differential equation that satisfies the initial condition $y(3) = 1$.

(g) What can you say about a solution of the differential equation $y' = -\frac{1}{2}y^2$ *just* by looking at the differential equation? Circle all possibilities.

1. The function y must be equal to 0 on any interval on which it is defined.
2. The function y must be strictly increasing on any interval on which it is defined.
3. The function y must be increasing (or equal to 0) on any interval on which it is defined.
4. The function y must be decreasing (or equal to 0) on any interval on which it is defined.
5. The function y must be strictly decreasing on any interval on which it is defined.

(h) Verify that all members of the family $y = \frac{2}{x+C}$ are solutions of the differential equation $y' = -\frac{1}{2}y^2$.

(i) Write a solution of the differential equation $y' = -\frac{1}{2}y^2$ that is not a member of the family $y = \frac{2}{x+C}$.

(j) Find a solution of the initial-value problem. $y' = -\frac{1}{2}y^2 \quad y(0) = 0.1$

(k) Find a solution of the initial-value problem. $y' = -\frac{1}{4}y^2 \quad y(0) = 0.2$

(l) Find a solution of the initial-value problem. $y' = -\frac{1}{3}y^2 \quad y(0) = 0.5$

(m) Find a solution of the initial-value problem. $y' = -\frac{1}{6}y^2 \quad y(0) = 0.5$

(n) A population is modeled by the differential equation

$$\frac{dP}{dt} = 1.1P\left(1 - \frac{P}{4000}\right)$$

1. For what values of P is the population increasing?
2. For what values of P is the population decreasing?
3. What are the equilibrium solutions?

(o) A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y^4 - 8y^3 + 15y^2.$$

1. What are the constant solutions of the equation?
2. Sketch the polynomial $t^4 - 8t^3 + 15t^2$. In particular, mark the x-intercepts.
3. For what values of y is y increasing?
4. For what values of y is y decreasing?

4. (9.1 Worksheet)

(a) True or false? Every differential equation has a constant solution. (If true, explain. If false, give a counterexample.)

(b) Consider the differential equation $\frac{dy}{dt} = 1 - 2y$.

i. Find all constant solution/s.

ii. Which of the following is a family of solutions? You may need to circle more than one.

$$y(t) = 1 + Ke^{-2t} \qquad y(t) = -Ke^{-2t} \qquad y(t) = \frac{1}{2} + Ke^{-2t} \qquad y(t) = \frac{1}{2} - Ke^{-2t}$$

5. (9.3 reading homework)

(a) Draw a rough sketch of a possible solution to the logistic differential equation $\dfrac{dP}{dt} = 5P\left(1 - \dfrac{P}{8}\right)$. **You do not need to solve this differential equation to draw a rough sketch.** Hint: Explained in https://www.khanacademy.org/math/ap-calculus-bc/bc-diff-equations/bc-logistic-models/e/logistic-differential-equation

6. (9.3 WebAssign)

(a) Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dx} = \frac{x}{y}, \quad y(0) = -9$$

(b) Find the solution of the differential equation that satisfies the given initial condition.

$$xy' + y = y^2, \quad y(1) = -8$$

(c) Consider the differential equation $(x^2 + 15)y' = xy$.

i. Find all constant solutions.

ii. Find all solutions.

(d) The differential equation below models the temperature of a 86^o C cup of coffee in a 20^o C room, where it is known that the coffee cools at a rate of 1^o C per minute when its temperature is 70^o C. Solve the differential equation to find an expression for the temperature of the coffee at time t. (Let y be the temperature of the cup of coffee in ^oC, and let t be the time in minutes, with $t = 0$ corresponding to the time when the temperature was 86^o C.)

$$\frac{dy}{dt} = -\frac{1}{50}(y - 20)$$

(e) A tank contains 8000 L of brine with 14 kg of dissolved salt. Pure water enters the tank at a rate of 80 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate.

1. How much salt is in the tank after t minutes?
2. How much salt is in the tank after 20 minutes?

(f) Find the orthogonal trajectories of the family of curves $y^2 = 8kx^3$. Sketch these orthogonal trajectories.

7. (Sec 10.3 Week 15 Quiz)

(a) Sketch the polar equation $r = \frac{5}{2}$

(b) Sketch the polar equation $\theta = \frac{\pi}{4}$

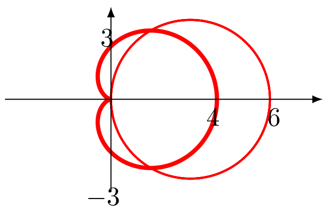
(c) Convert the polar equation $r = 3$ to Cartesian.

(d) Convert the polar equation $\theta = \frac{\pi}{3}$ to Cartesian.

(e) Convert the polar equation $\theta = \frac{\pi}{6}$ to Cartesian.

(f) Convert the polar equation $r = 9\cos\theta$ to Cartesian.

8. Consider the circle $r = 6\cos\theta$ and the cardioid $r = 2 + 2\cos\theta$.



(a) Mark points on *both* curves where $\theta = 0, \dfrac{\pi}{4}$, and $\dfrac{\pi}{2}$.

(b) Shade in the area inside the circle and outside the cardioid.

(c) Find the area (which you shade) inside the circle and outside the cardioid.

9. (Sec 10.1, 10.2 Week 12 quiz)

i. (a) Find *parametric* equations for the top half of the circle centered at $(2, 3)$ with radius 5, oriented *clockwise*.

$x =$ ______________________

$y =$ ______________________

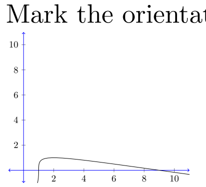
for ____________ $\leq t \leq$ ___________

(b) Eliminate the parameter to find a Cartesian equation of the curve.

ii. Consider the curve described by the parametric equations

$$x = t^3 + 1$$
$$y = 2t - t^2, \qquad \text{for } -\infty < t < \infty$$

(a) Mark the orientation on the curve (direction of increasing values of t).



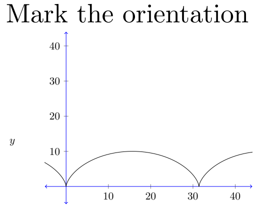
(b) Find the area enclosed by the x-axis and the given curve.

(c) Perform and describe a reality check by comparing your answer and the graph which has been drawn to scale.

iii. Consider the cycloid which is described by the parametric equations

$$x = 5(t - \sin t)$$
$$y = 5(1 - \cos t), \qquad \text{for } \infty < t < \infty$$

(a) Mark the orientation on the curve (direction of increasing values of t).



(b) Find the area enclosed by the x-axis and *one* arch of the cycloid. Hint: $dx = 5(1 - \cos t)\ dt$.

(c) Perform a reality check by comparing your answer and the graph (which is drawn to scale).

10. (Sec 11.10)

 i. *If* f has a power series representation at 4, that is, if $f(x) = \sum_{n=0}^{\infty} c_n (x-4)^n$ for $|x-4| < R$, then its coefficients are given by the formula $c_n =$ ________________________. Answer: Theorem 5 on page 760.

 Give a proof for your formula for c_n. Answer: follow pg 759 or lecture notes https://egunawan.github.io/fall17/notes/notes11_10part1.pdf.

 ii. Circle all the true statements and cross out all the false statements, and justify.

 (a) If the series $\sum_{n=1}^{\infty} c_n x^n$ converges for $|x| < R$, then $\lim_{n\to\infty} c_n x^n = 0$ for $|x| < R$.

 (b) If the series $\sum_{n=1}^{\infty} c_n x^n$ diverges for $x = 5$, then $\lim_{n\to\infty} c_n x^n \neq 0$ for $x = 5$.

 iii. Find the Maclaurin series for $f(x) = 6(1-x)^{-2}$ using the definition of a Maclaurin series. (You may assume that $f(x)$ has a power series expansion). Find the associated radius of convergence. Use Ratio Test to find the radius of convergence $\boxed{R = 1}$.

 iv. Use a Maclaurin series given in this table `http://egunawan.github.io/fall17/quizzes/11_10_table01.pdf` (printed on the next page) to obtain the Maclaurin series for the function $f(x) = 8e^x + e^{8x}$. Find the radius of convergence.

 v. Evaluate the indefinite integral $\left(8 \int \frac{e^x - 1}{5x} \, \mathrm{dx}\right)$ as an infinite series.

 vi. Find the Maclaurin series for $f(x) = e^{-4x}$ using the definition of a Maclaurin series. Don't use the table. (You may assume that $f(x)$ has a power series expansion). Find the associated radius of convergence R.

11. (11.3 Integral test and p-series, 11.8 geometric series test/ratio test to find interval of convergence, 11.9 power series representation of a function, Week 8 quiz)

12. (Section 11.3) Suppose f is a continuous, positive, and decreasing function on $[1, \infty)$ and $a_k = f(k)$. By drawing a picture, rank the following three quantities in increasing order:

$$\int_1^6 f(x) \, \mathrm{dx} \qquad \sum_{k=1}^{5} a_k \qquad \sum_{k=2}^{6} a_k$$

13. (Sec 11.3 p-series) The *Riemann zeta*-function ζ ("zeta") is defined by

$$\zeta(x) := \sum_{n=1}^{\infty} \frac{1}{n^x}.$$

It is used in number theory to study the distribution of prime numbers.

 (a) What is the domain of the function ζ? (That is, for what values of x is this function defined?)

 (b) Euler computed $\zeta(2)$ to be $\frac{\pi^2}{6}$. (See page 720, sec 11.3). Use this fact to find the sum of each series below.

$$\sum_{n=3}^{\infty} \frac{1}{n^2} \qquad \sum_{n=1}^{\infty} \frac{1}{(5n)^2} \qquad \sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$$

14. (Section 11.8 power series)

 (a) What is a power series? (See Sec 11.8, top of page 747)

 (b) In most cases, how do you find the radius of convergence of a power series?

 (c) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}.$$

Answer: see Example 5, pg 750.

(d) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} n! x^{2n}. \qquad \text{Answer: see Example 1, pg 747.}$$

(e) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{n^5}. \text{ Answer: same } R \text{ as Ex 2, pg 747, but both endpoints are included.}$$

(f) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{n!}. \qquad \text{Answer: same answer as Example 3, pg 748.}$$

15. (Sec 11.9 WebAssign finding interval of convergence) For each function, find a power series representation and determine the interval of convergence.
(Check your work with WolframAlpha. Type "series representation of ...")
   (a) $f(x) = \dfrac{1}{3+x}$ (see Sec 11.9 Example 2)
   (b) $f(x) = \dfrac{x^3}{5+x}$ (see Sec 11.9 Example 3)
   (c) $f(x) = \dfrac{x}{1+10x^2}$ (a variation of Sec 11.9 Example 3)

16. (Sec 11.9 WebAssign differentiation and integration of power series) For each function, find a power series representation. Determine the radius of convergence. (You do not need to determine the interval of convergence)
   (a) $f(x) = \dfrac{1}{(2+x)^2}$ (a variation of Sec 11.9 Ex 5)
   (b) $f(x) = \ln(1+x)$ (see Sec 11.9, Ex 6)
   (c) $f(x) = arctan(x)$ (see Sec 11.9, Ex 7)
   (d) $\displaystyle\int \frac{1}{1+x^7}$ dx (see Sec 11.9, Ex 8)
   (e) $\displaystyle\int \frac{x}{1-x^7}$ dx (a variation Sec 11.9, Ex 8)

17. (Ch 7 Integration methods) Decide whether the best method of integration is integration by parts, u-substitution, or trig substitution. Explain the first key step/s of evaluating the integrals (There often are more than one right answer).
   (a) $\displaystyle\int_0^4 \frac{\ln(x)}{\sqrt{x}}$ dx ____________________
   (b) $\displaystyle\int \frac{1}{x\ln(x)}$ dx ____________________
   (c) $\displaystyle\int_1^2 \ln(x)$ dx ____________________
   (d) $\displaystyle\int xe^{0.2x}$ dx ____________________
   (e) $\displaystyle\int_0^1 e^x \sin(x)$ dx ____________________
   (f) $\displaystyle\int \frac{1}{x^2+2x+4}$ dx ____________________

18. Evaluate $\int \frac{1}{x^2+25}$ dx

19. (Section 7.4 WebAssign partial fraction decomposition)

(a) Evaluate $\int \frac{10}{(x+5)(x-2)}$ dx

(b) Evaluate $\int \frac{x+4}{x^2+2x+5}$ dx

(c) Evaluate $\int \frac{7x^2-6x+16}{x^3+4x}$ dx

20. (7.8 improper integral, 11.3 integral test)

i. (a) Find the values of p for which the integral $\int_e^\infty \frac{6}{x(\ln x)^p}$ dx converges. Evaluate the integral for these values of p.

(b) Determine whether

$$\int_2^\infty \left(\frac{1}{e^5}\right)^x \text{ dx}$$

is convergent or divergent. If it is convergent, evaluate it.

(c) Determine whether

$$\int_2^\infty \frac{1}{x^2+8x-9} \text{ dx}$$

is convergent or divergent. If it is convergent, evaluate it.

(d) Determine whether $\int_0^1 4x^{-5}$ dx is convergent or divergent. If it is convergent, evaluate it.

(e) Determine whether

$$\int_0^1 \frac{4}{x^{0.5}} \text{ dx}$$

is convergent or divergent. If it is convergent, evaluate it.

(f) Determine whether

$$\int_2^3 \frac{2}{\sqrt{3-x}} \text{ dx}$$

is convergent or divergent. If it is convergent, evaluate it.

ii. (a) Evaluate the integral $\int_1^\infty \frac{3}{x^6}$ dx. Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series $\sum_1^\infty \frac{3}{n^6}$ is convergent or divergent.

(b) Evaluate the integral

$$\int_1^\infty \frac{1}{(4x+2)^3} \text{ dx.}$$

Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series

$$\sum_1^\infty \frac{1}{(4n+2)^3}$$

is convergent or divergent.

(c) Evaluate the integral $\int_1^\infty \frac{1}{\sqrt{x+9}}$ dx. Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series

$$\sum_1^\infty \frac{1}{\sqrt{n+9}}$$

is convergent or divergent.

(d) Evaluate the integral $\int_1^{\infty} x\, e^{-9x}$ dx. Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series $\sum_1^{\infty} ne^{-9n}$ is convergent or divergent.

(e) Are the conditions for the Integral Test satisfied for the series

$$\sum_1^{\infty} (\cos\ n)^2 \ +\ \frac{1}{n}\ \text{dx}\ \ ?$$

(f) Are the conditions for the Integral Test satisfied for the series

$$\sum_1^{\infty} \frac{(\cos\ n)^2}{n}\ \text{dx}\ \ ?$$

21. (Week 4 quiz)

i. (a) Consider $\sum \frac{5^n}{n!}$ and $\sum \frac{n!}{n^n}$ (similar to Sec 11.6 Example 5 pg 741).

(b) Show your work for attempting to use the ratio test to this series to determine whether it converges or diverges.

(c) If the ratio test is conclusive, determine whether the series is convergent or divergent. Otherwise, state that the ratio test is inconclusive.

ii. Determine whether the series $\sum_{n=1}^{\infty} \frac{\ln\ n}{n}$ and $\sum_{n=4}^{\infty} \frac{1}{2^n-9}$ (from Sec 11.4 Examples 2 and 3) converge

iii. Determine whether the series $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}}$ and $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ (from Sec 11.4 Examples 1 and 4) converge.

22. (11.2: geometric and harmonic series, 11.4: comparison test, 11.6: ratio tests) STRATEGY TIPS:

✓ The ratio test usually works when the term contains factorial like $(n+3)!$ or exponents like 7^n, $\frac{1}{7^n}$.

✗ The ratio test will *not* work with series with ONLY p-series-like terms, for example, $\sum \frac{n^2+4}{\sqrt{n^5-1}}$. Convince yourself.

✓✗ Use one of the comparison tests are when the series looks like the geometric series $\sum r^n$, or the p-series $\sum \frac{1}{n^p}$.

Show whether each series $\sum a_n$ converges. For full credit you should give

- The series $\sum b_n$ and an explanation why $\sum b_n$ converges/diverges (if you use Comparison or Limit Comparison test)
- An inequality or limit computation
  - If using the Comparison Test, give an inequality of the form $a_n \leq b_n$ or $a_n \leq b_n$
  - If using the Limit Comparison Test, compute $\lim_{n\to\infty} a_n/b_n$
- A conclusion statement.

i. (Book examples) Pg 728-730: Sec 11.4 Ex 1,2,3,4; Pg 740-741 Sec 11.6 Ex 3, 5

ii. (a) $\sum_{n=2}^{\infty} \dfrac{n^3}{n^4+1}$

(b) $\sum_{n=1}^{\infty} \dfrac{6^n}{5^n-1}$

(c) $\sum_{n=1}^{\infty} \dfrac{(2n-1)(n^2-1)}{(n+1)(n^2+4)^2}$

iii. (Sec 11.4)

(a) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{n+6^n}{n+2^n}$ converges or diverges. ✓LCT attempt 1: You try LCT with $\sum(\frac{6}{2})^n$ and it works.
✓LCT attempt 2: LCT with $\sum \frac{1}{n}$ also works. But this may not be the first thing that comes to your mind.
✓Divergence test: the terms are increasing, so this test works.
✓Comparison test: find a big enough constant A so that $a_n > A(\frac{6}{2})^n$ - see WebAssign solution.
✓Ratio test: you see powers, so you try the ratio test. The ratio $\frac{a_{n+1}}{a_n}$ goes to 6/2.

(b) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{2n+3^n}{2n+7^n}$ converges or diverges. ✓LCT attempt 1: You try LCT with $\sum(\frac{3}{7})^n$ and it works.
✓LCT attempt 2: LCT with $\sum \frac{1}{n^2}$ also works, but this may not be the first thing that comes to your mind.
✗Divergence test: inconclusive.
✓Comparison test: find a big enough constant A so that $a_n < A(\frac{3}{7})^n$ - see WebAssign solution.
✓Ratio test: you see powers, so you try the ratio test. The ratio $\frac{a_{n+1}}{a_n}$ goes to 3/7.

(c) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{5n^2-1}{6n^4+7}$ converges or diverges. (Hint: compare with a p-series and use one of the comparison tests. Would the ratio test be conclusive? See strategy above.)

(d) Determine whether the series $\sum_{n=6}^{\infty} \dfrac{n-5}{n7^n}$ converges or diverges. (Hint: compare with a geometric series. You can also try the ratio test because you see powers $(\frac{1}{7})^n$)

(e) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{5^n}{n7^n}$ converges or diverges. (Hint: compare with a geometric series. You can also try the ratio test because you see powers $(\frac{5}{7})^n$)

(f) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{5^{2n}}{n7^n}$ converges or diverges.

(g) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{n+8}{n\sqrt{n}}$ converges or diverges.

iv. (Divergence Test Sec 11.2)

(a) True or false? If a_n does not converge to 0, then the series of $\sum_{n=1}^{\infty} a_n$ diverges.
(b) True or false? If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
(c) Let $a_n = \frac{4n}{7n+1}$. (I) Determine whether $\{a_n\}$ is convergent. (II) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.
(d) Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2-1}{100+5n^2}$ is convergent or divergent.

v. (Sec 11.6)

(a) $\sum_{n=1} \dfrac{5n!}{2^n}$ (hint: see factorial, think ratio test)

(b) $\sum_{n=1} \dfrac{n}{5^n}$ and $\sum_{n=1} ne^{-5n}$ ✗LCT with geom. series $\sum(\frac{1}{5})^n$ is inconclusive.
✓LCT comparing with $\sum \frac{1}{n^2}$ works.
✓You see power $(\frac{1}{5})^n$, so you try ratio test.

(c) $\sum_{n=1} \left(\dfrac{1}{4n+1}\right)^n$ ✓LCT: compare with p-series like $\sum \frac{1}{n^2}$.
✓LCT: compare with geometric series like $\sum \frac{1}{4^n}$.
✓Can use ratio test because you see powers something^n, but the computation for the ratio test is long.

(d) $\sum_{n=1} n\left(\dfrac{5}{7}\right)^n$

(e) $\sum_{n=1} \dfrac{|\sin(5n)|}{5^n}$ ✓see sin and $(\frac{1}{5})^n$, so think comparison test with the geometric series $\sum(\frac{1}{5})^n$.
✗The LCT with $b_n = (\frac{1}{5})^n$ fails.
✓The LCT with $b_n = \frac{1}{n^2}$ works.
✗Ratio test fails.

(f) $\sum_{n=1} \dfrac{|\sin(5n)|}{n^5}$ ✓see sin and $(\frac{1}{n^5})$, so think the (non-limit) comparison test with the p-series $\sum(\frac{1}{n^5})$.
✗The LCT with $b_n = (\frac{1}{n^5})$ fails.
✓The LCT with $\frac{1}{n^3}$ works.
✗Ratio test fails.

(g) $\sum_{n=1} \dfrac{2^n}{n^3}$. ✓Divergence test: numerator grows faster than the denominator, so use divergence test.
✗LCT: you see 2^n, but find that the comparison tests with the geometric series $\sum 2^n$ are inconclusive.
✓LCT: you try LCT with $\sum \frac{1}{n}$ and find that it works.
✓Ratio test: you can try ratio test because you see power 2^n.
✓Root test (not required to memorize): you can try root test because you see power 2^n.

(h) $\sum_{n=1} \dfrac{n!}{100^n}$ ✓Ratio test: you see *factorial* and exponent 100^n, so think ratio test.
✓Divergence test: you remember than factorial grows faster than exponential.
✗LCT: You try comparing it with $\sum n!$ but the result is inconclusive.

(i) $\sum_{n=1} \dfrac{n}{\sqrt{n^3+4}}$ ✓LCT or comparison: looks like a p-series, so use either.
✗Ratio test: you attempt the ratio test, and you get an inconclusive result. But I've told you above that the ratio test will not work for any series that look like a p-series.)

23. (Sec 11.1)

i. (a) Compute $\lim_{n\to\infty}\left(1+\dfrac{1}{2n}\right)^n$ and $\lim_{n\to\infty}\left(1+\dfrac{5}{4n}\right)^n$ if they exist. (Hint: Note the indeterminate form "1^∞".)

(b) Determine whether the sequence $\left\{\dfrac{5n!}{2^n}\right\}_{n=1}^{\infty}$ converges or diverges.

ii. Each of the following series a_n converges to a limit L. Given $\epsilon > 0$, find *a* positive number N such that, if $n > N$, then a_n is within distance ϵ of L. SEE EXAMPLE https://egunawan.github.io/fall17/notes/notes11_1choosingN.pdf

(a) $a_n = \frac{1}{n^2+3}$, $L = 0$.

(b) $a_n = \frac{3n+2}{2n-1}$, $L = \frac{3}{2}$.

(c) $a_n = \frac{n^2+2}{n^2-3}$, $L = 1$.

iii. The sequence $a_n = (5n+4)/(3n^2-2)$ converges to 0. Justify this fact by squeezing a_n between 0 and another sequence $b_n =$ (a constant)$/n$ and using the Squeeze theorem. You may assume $\lim_{n\to\infty}$ (any constant)$/n = 0$.
See a similar (model) solution at: https://egunawan.github.io/fall17/hw/models.pdf

iv. Either give an example satisfying the condition or explain why no such sequence exists.

(a) A monotonically decreasing sequence that converges to 10.

(b) A monotonically increasing bounded sequence that does not converge (that is, diverges).

(c) A non-monotonic sequence that converges to 4.

24. (Sec 11.2 telescoping sum)

(a) (i.) Find a formula for the n-th partial sum S_n of the series $\sum_{k=2}^{\infty} \dfrac{5}{k^2-1}$.

(ii.) Evaluate $\lim_{n\to\infty} S_n$.

(iii.) Use the previous part to determine the sum of the series $\sum_{k=2}^{\infty} \dfrac{5}{k^2-1}$ or state that the series diverges.

(b) (i.) Find a formula for the n-th partial sum S_n of the series $\sum_{k=1}^{\infty} \frac{2}{\sqrt{k}} - \frac{2}{\sqrt{k+3}}$.

(ii.) Evaluate $\lim_{n\to\infty} S_n$.

(iii.) Use the previous part to determine the sum of the series $\sum_{k=1}^{\infty} \frac{2}{\sqrt{k}} - \frac{2}{\sqrt{k+3}}$ or state that the series diverges.

25. (Sec 11.2 geometric series and decimal expansion)

(a) Use geometric series to write $2.74\bar{9} = 2.74999999\ldots$ as a fraction.

(b) Write a *different* decimal expansion for $2.74999999\ldots$.

(c) Write the repeating decimal expansion $0.\overline{571428}$ as a fraction.

(d) (i) Determine whether the series $\sum_{k=1}^{\infty} 2^{2k}3^{1-k}$ is convergent. (ii) If it is convergent, compute the sum.

(e) (i) Determine whether the series

$$\sum_{k=1}^{\infty} 2^{2k}3^{5-2k} + 7^{k+1}10^{-k} + 3^{k+1}4^{-k}$$

is convergent or divergent. (ii) If it is convergent, compute the sum.

(f) (i) Find a closed-form formula for the nth term in the sequence $\left\{3, -4, \dfrac{16}{3}, -\dfrac{64}{9}, \ldots\right\}$. (ii) Use it to determine whether the series

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \ldots$$

is convergent or divergent. (iii) If it is convergent, compute the sum.

(g) (i) Determine whether the series $\sum_{k=1}^{\infty} \frac{(-1)^n}{e^{2n-1}}$ is convergent or divergent. (ii) If it is convergent, compute the sum.