MATH 1152 Exam 2 Summary Chapter 11

[Ch 11] Power Series

【11.3】The Integral Test

1. Use the **Integral Test** to determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ is convergent or divergent.

2. (**Not in Exam 2**) Consider the series $\sum_{n=1}^{\infty} e^{-n}$. It can be shown that the series converges by applying the Integral Test. Assume that the conditions for the Integral Test have been verified. Use the Remainder Estimate for the Integral Test to bound (lower and upper) the error when the sum of the series is approximated by adding the first 7 terms.

【11.5】Alternating Series

1. Use the **Alternating Series Test** to determine the convergence or divergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$.

2. Determine whether the series $\sum_{n=1}^{\infty} \cos(n\pi) \tan\left(\frac{\pi}{n}\right)$ is convergent or divergent.

3. If the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$ converges, use the Alternating Series Estimation Theorem to provide a bound on the absolute value of the error, when the sum of the series is approximated by adding up the first 4 terms.

4. Determine how many terms of the convergent series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ must be summed to be sure that the remainder is less than 10^{-4}?

【11.8】Power Series

1. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x=-4$ but diverges when $x=6$. Determine if the series is convergent of divergent.

   a. $\sum_{n=0}^{\infty} c_n$

   b. $\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

2. Find the radius of convergence and the interval of convergence for the power series.

   a. $\sum_{n=1}^{\infty} \frac{n^{20} x^n}{(2n+1)!}$

   b. $\sum_{n=0}^{\infty} \frac{n!(x-2)^n}{3^n}$

   c. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{7^n (n+1)}$

3. Find the radius of convergence and the interval of convergence for the power series $\sum_{n=2}^{\infty} \frac{1}{n \ln n} \left(\frac{x}{2} - 1 \right)^n$. Hint: to test the endpoints, use #1 in 【11.3】 and #1 in 【11.5】.

【11.9】Representations of Functions

1. Consider the function $f(x) = \frac{3}{x^2 - x - 2}$.

   a. Express $f(x)$ as a power series by first using partial fraction decomposition.

   b. Find the interval of convergence for the power series in part(a).

2. It is known that $\frac{1}{1-x}$ has a power series representation $x=1$ for $|x|<1$.

   a. Find a power series representation for $\frac{1}{1+x^5}$.

   b. Find a power series representation for $\int \frac{1}{1+x^5}\, dx$.

c. Assume that the series you found in part (b) converges for $|x|<1$. Use your answer in part (b) to determine a series that represents $\int_0^{0.2} \frac{1}{1+x^5}\,dx$.

d. If the first two non-zero terms of the series are used to estimate the value of the definite integral from part (c), provide a bound on the error of this estimate.

3. It is known that e^x has a power series representation $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ for $-\infty < x < \infty$.

a. Find a power series representation for xe^x.

b. Find a power series representation for $\frac{d}{dx}\left(xe^x\right)$.

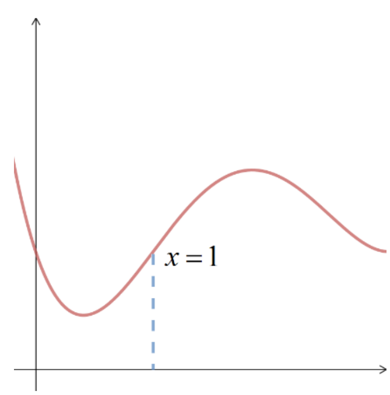
c. Assume that the series you found in part (b) converges for $-\infty < x < \infty$. Use your answer in part (b) to evaluate $\sum_{n=0}^{\infty} \frac{(n+1)(-1)^n}{n!}$.

d. Find a power series representation for $\int xe^x\,dx$.

e. Assume that the series you found in part (d) converges for $-\infty < x < \infty$. Use your answer in part (d) to evaluate $\sum_{n=0}^{\infty} \frac{1}{(n+2)n!}$.

【11.10】Taylor and Maclaurin Series

1. The graph of f is shown.



a. Explain why the series

$1-0.8(x-1)+0.5(x-1)^2-0.3(x-1)^3+$

is **NOT** the Taylor series of f centered at $x=1$.

b. Explain why the series

$1+0.6(x-2)+0.5(x-2)^2-0.4(x-2)^3+$

is **NOT** the Taylor series of f centered at $x=2$.

2. Find the Taylor series for the function $f(x)$ centered at a. In addition, find the interval of convergence.

a. $f(x)=x^4-3x^2+1,\ a=1$

b. $f(x)=\ln x,\ a=2$

3. Consider the function $f(x) = \ln(1+x)$.
    a. Find the Maclaurin series for $f(x)$.
    b. Use part (a) to evaluate $\lim_{x \to 0} \frac{x - \ln(1+x)}{x^2}$.

【11.11】Applications of Taylor Polynomials

1. Consider the function $f(x) = \frac{1}{1+x}$.
    a. Approximate $f(x)$ by the Taylor polynomial $T_2(x)$ centered at $a = 0$.
    b. Approximate $\frac{1}{1.05}$.

2. Consider the function $f(x) = \sin x$.
    a. Approximate $f(x)$ by the Taylor polynomial $T_4(x)$ centered at $a = \frac{\pi}{6}$.
    b. Use Taylor's Inequality to provide a bound on the error of this estimate when *x* is between *0* and *pi/3*.

MATH 1152Q Exam 2 Summary Ch 11 Answer

[Ch 11] Power Series

【11.3】The Integral Test

(1) The series diverges. (2) $\frac{1}{e^8} \le R_7 \le \frac{1}{e^7}$

【11.5】Alternating Series

(1) The series converges. (2) The series converges. (3) $|R_4| \le \frac{5}{26}$ (4) 10

【11.8】Power Series

(1) (a) The series is convergent. (b) The series is divergent.

(2) (a) The interval of convergence is $(-\infty, \infty)$ and the radius of convergence is ∞.

(b) The series converges only when $x = 2$, in which case the radius of convergence is 0.

(c) The interval of convergence is $(-7, 7]$ and the radius of convergence is 7.

(3) The interval of convergence is $[0, 4)$ and the radius of convergence is 2.

【11.9】Representations of Functions

(1) (a) $\sum_{n=0}^{\infty}\left[(-1)^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right]x^n$ (b) $(-1,1)$ (2) (a) $\sum_{n=0}^{\infty}(-1)^n x^{5n}$ (b) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{5n+1}}{5n+1} + C$

(c) $\sum_{n=0}^{\infty}(-1)^n \frac{(0.2)^{5n+1}}{5n+1}$ (d) $\frac{(0.2)^{11}}{11}$ (3) (a) $\sum_{n=0}^{\infty}\frac{x^{n+1}}{n!}$ (b) $\sum_{n=0}^{\infty}\frac{(n+1)x^n}{n!}$ (c) 0 (d) $\sum_{n=0}^{\infty}\frac{x^{n+2}}{(n+2)n!} + C$ (e) 1

【11.10】Taylor and Maclaurin Series

(1) (a) $f'(1) = -0.8 < 0$ contradicts the graph. (b) f''(2)=1>0 contradicts the graph.

(2) (a) $-1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + (x-1)^4$, the interval of convergence is $(-\infty, \infty)$

(b) $\ln 2 + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n2^n}(x-2)^n$, the interval of convergence is $(0, 4]$ (3)(a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$ (b) $\frac{1}{2}$

【11.11】Applications of Taylor Polynomials

(1) (a) $1-x+x^2$ (b) 0.9525 (2) (a) $\frac{1}{2}+\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{6}\right)-\frac{1}{4}\left(x-\frac{\pi}{6}\right)^2-\frac{\sqrt{3}}{12}\left(x-\frac{\pi}{6}\right)^3+\frac{1}{48}\left(x-\frac{\pi}{6}\right)^4$

(b) $\frac{1}{5!}\left(\frac{\pi}{6}\right)^5$