1. Let $f(x)$ be continuous everywhere.

   (a) $\int_a^\infty f(x)\ \mathrm{dx} = \lim_{t\to\infty} \int_a^t f(x)\ \mathrm{dx}$ **T** **F**

   Justification:

   (b) $\int_a^\infty f(x)\ \mathrm{dx} = \lim_{a\to\infty} \int_a^t f(x)\ \mathrm{dx}$ **T** **F**

   Justification:

   (c) True of false? $\int_0^a f(x) = \int_0^a f(a-x)\ \mathrm{dx}$. If T, justify. If F, give a counterexample.

   (d) True of false? $\int_0^a f(x) = \int_0^a f(x-a)\ \mathrm{dx}$. If T, justify. If F, give a counterexample.

   (e) Write a sanity-check-type calculation (different from what you've written) to further confirm your answer in the previous two questions.

2. Evaluate the integral if possible. If they diverge, state that it diverges.

   (a)
   $$\int_0^1 \frac{27}{x^5}\ \mathrm{dx}$$

   (b)
   $$\int_{-1}^1 \frac{1}{x}\ \mathrm{dx}$$

3. **Multiple choice**

   i.) If $\lim_{x\to\infty} f(x) = 0$, then $\lim_{x\to\infty} (xf(x))$ is ...
   (a) zero (b) ∞ (c) non-zero constant (d) another method is needed to determine this

   ii.) If $\lim_{x\to\infty} f(x) = \infty$, then $\lim_{x\to\infty} (f(x) - x)$ is ...
   (a) zero (b) ∞ (c) non-zero constant (d) another method is needed to determine this

   iii.) If $\lim_{x\to 0} f(x) = 0$, then $\lim_{x\to 0} (f(x) - x)$ is ...
   (a) zero (b) ∞ (c) non-zero constant (d) another method is needed to determine this

   iv.) If $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = \infty$, then $\lim_{x\to 0} (f(x) + g(x))$ is ...
   (a) zero (b) ∞ (c) non-zero constant (d) another method is needed to determine this

   v.) If $\lim_{x\to\infty} f(x) = 1$, then $\lim_{x\to\infty} (f(x))^x$ is ...
   (a) zero (b) ∞ (c) 1 (d) another method is needed to determine this

4. (not in Exam 2) Suppose that $f(1) = 3$,$f(4) = 8$, $f'(1) = 5$, $f'(4) = 5$ and f'' is continuous. Find the value of
   $$\int_1^4 xf''(x)\ \mathrm{dx}.$$

5. (from 7.4 WebAssign, also in Week 7 sample quiz)

(a) Decompose the following into partial fractions:

$$\frac{3x^2 + 2x - 3}{x^3 - x}$$

(b) Provide a computation that is either a formal verification (that is, a proof) or simply a sanity-check (not a valid proof) for your answer to the previous question.

(c) Evaluate $\int \frac{10}{(x+5)(x-2)}$ dx.

(d) Provide a computation that is either a formal verification (that is, a proof) or simply a sanity-check for your answer to the previous question.

6. For the following questions, circle TRUE or FALSE. Justify briefly. (Hint: Section 11.5)

(a) An application of the Alternating Series Estimation Theorem is the proof that e is irrational.
T **F**

(b) An application of the Alternating Series Estimation Theorem is the proof that e is rational.
T **F**

(c) An application of the Alternating Series Estimation Theorem is a way to ensure that we can get an approximation to the *definite* integral $\int_0^1 e^{-x^2}$ dx using series so that the approximation is within a certain error bound (for example 1/1000). **T** **F**

(d) We can always use the Alternating Series Estimation Theorem to ensure that we can get an approximation of a *function* using its Taylor polynomial so that the approximation is within a certain error bound (for example, 1/1000) on a certain interval. **T** **F**

(e) Suppose $b_k > 0$ for all k and $\sum_{k=1}^{\infty}(-1)^k\, b_k$ is a convergent with sum S and partial sum S_n. Then $|S - S_5| \le b_6$. **T** **F**

(f) Suppose $b_k > 0$ for all n and $\sum_{k=1}^{\infty}(-1)^k\, b_k$ is a convergent with sum S and partial sum S_n. Then $|S - S_5| \ge b_6$. **T** **F**

7. (From WebAssign 11.8 no. 4)

(a) Suppose that the radius of convergence of the power series $\sum c_n x^n$ is 16. What is the radius of convergence of the power series $\sum c_n x^{4n}$?

(b) Suppose that the radius of convergence of the power series $\sum c_n x^n$ is R. What is the radius of convergence of the power series $\sum c_n x^{5n}$?

(c) The integral $\int_2^3 \sqrt{x-2}$ dx is improper. **T** **F**

(d) When is an integral improper? Hint: There are two kinds. See Sec 7.8 pg 527 and 531.

(e) Write three definite improper integrals.

(f) When is an integral proper? (The answer is when it is not improper, but explain what needs to happen for a definite integral to be proper).

(g) Write three definite proper integrals.

8. (a) If the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n x^n$ is [-9,11), what is the radius of convergence of the series $\sum_{n=1}^{\infty} n\, c_n x^{n-1}$? Why?

(b) If the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n x^n$ is [-9,11), what is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{c_n}{n+1} x^{n+1}$? Why?

(c) Use Table 1 to show that $\frac{d}{dx}\cos(x) = -\sin(x)$.

(d) Write the first three nonzero terms of the Maclaurin series for $\tan(x)$ using Table 1 and long division of power series.

(e) Use the series that you just computed for $\tan(x)$ to evaluate

$$\lim_{x\to 0} \frac{\tan(x) - x}{x^3}.$$

(f) Use a different method to evaluate

$$\lim_{x\to 0} \frac{\tan(x) - x}{x^3}.$$

(g) Write the first three nonzero terms of the Maclaurin series for $e^x \sin(x)$ using Table 1 and multiplication of power series.

(h) Write the first three nonzero terms of the Maclaurin series for $\sec(x)$ using long division of power series and Table 1.

9. (from quiz week 8)

(a) What is a power series?

(b) In most cases, how do you find the radius of convergence of a power series?

(c) Find the *interval* of convergence of the following series.

- $\sum_{n=0}^{\infty} \frac{n\,(x-5)^n}{3^{n+1}}$. Answer: see Example 5, pg 750.
- $\sum_{n=0}^{\infty} n!\,(x-5)^n$. Answer: see Example 1, pg 747.
- $\sum_{n=0}^{\infty} \frac{(x-5)^{\frac{n}{2}}}{n}$. Answer: same radius of convergence as Example 2, pg 747.
- $\sum_{n=0}^{\infty} \frac{(x-5)^n}{n!}$. Answer: same answer as Example 3, pg 748.

10. (from quiz week 8) (i) Without using the table, find the power series representations of $\ln(1+x)$, $\arctan(x)$, and $\int \frac{x}{1-x^5}$ dx. Use the table to verify your answer.
(ii) Determine the *radius* of convergence and *interval* of convergence.
(iii) Write the first 5 non-zero terms of the resulting power series.

11. (from Sec 11.10 homework) Find the Taylor series for $f(x) = \sqrt{x}$ centered at 9.

12. (From In-class worksheet on Week 9 Friday `https://egunawan.github.io/fall17/notes/1132q11_10_taylor_and_maclaurin_series.pdf`)
    (a) True or False? If $f(x) = 1 + 3x - 2x^2 + 5x^3 + \dots$ for $|x| < 1$ then $f'''(0) = 30$.
    (b) Can you write a Maclaurin series for $f(x) = \sqrt[3]{x}$? Explain why or why not.

13. Compute a power series representation for $\ln(1-x)$ using at least two different ways. (Check your answer with a technology).
    (a) (Way 1:)
    (b) (Way 2:)
    (c) Show at least two ways you can check your answer to this problem (without technology). The check could be a full verification or just a sanity check (not a full verification).
        1. Check 1
        2. Check 2

14. Use Maclaurin series to write the binomial series expansion of $(1+x)^6$.
Use Maclaurin series to write the binomial series expansion of $(1+x)^k$ where k is a positive integer.

15. Using Table 1 (series), prove that

$$Re^{i\theta} = R\cos\theta + iR\sin\theta$$

16. True or False.
    (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$ **T** **F**
    Justification:
    (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = -e$ **T** **F**
    Justification:
    (c) If $f(x) = 2x - x^2 + \frac{1}{3}x^3 - \dots$ converges for all x, then $f'''(0) = 2$ **T** **F**
    Justification:
    (d) If $f(x) = 2x - x^2 + \frac{1}{3}x^3 - \dots$ converges for all x, then $f'''(0) = \frac{1}{2}$ **T** **F**
    Justification:

17. (similar to WebAssign Sec 11.11)
    (a) Find the 3rd degree Taylor polynomial of the function $f(x) = xe^{-2x}$ centered at 0. Sketch this polynomial. Label all the important points - label at least four convenient points.
    (b) (i) Approximate $f(x) = \ln(1+2x)$ by a Taylor polynomial of degree 3 centered at 1.
    (ii) Use Taylor's Inequality to estimate the accurate of the approximation $T_3(x)$ of $f(x)$ when $0.8 \le x \le 1.2$. You do not need to simplify your answer.

(c) Approximate $f(x) = e^{4x^2}$ by a Taylor polynomial with degree 3 centered at 0.

18. (Integral Test from 11.3 WebAssign, also in Week 7 sample quiz)

(a) What are the conditions needed to apply the Integral Test ?

(b) Find the values of p for which the integral $\int_e^\infty \frac{6}{x(\ln x)^p}$ dx converges. Evaluate the integral for these values of p.
(Hint: Check what happens when $p = 1$, when $p < 1$, and when $p > 1$.)

(c) Evaluate the integral

$$\int_1^\infty \frac{3}{x^6} \text{ dx.}$$

Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series

$$\sum_1^\infty \frac{3}{n^6}$$

is convergent or divergent.

(d) Evaluate the integral

$$\int_1^\infty \frac{1}{(4x+2)^3} \text{ dx.}$$

Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series

$$\sum_1^\infty \frac{1}{(4n+2)^3}$$

is convergent or divergent.

(e) Evaluate the integral

$$\int_1^\infty \frac{1}{\sqrt{x+9}} \text{ dx.}$$

Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series

$$\sum_1^\infty \frac{1}{\sqrt{n+9}}$$

is convergent or divergent.

(f) Evaluate the integral

$$\int_1^\infty x\ e^{-9x} \text{ dx.}$$

Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series $\sum_1^\infty ne^{-9n}$ is convergent or divergent.

(g) Evaluate the integral

$$\int_1^\infty x\ e^{-9x} \text{ dx}$$

Are the conditions for the Integral Test satisfied? If so, use the Integral Test to determine whether the series

$$\sum_1^\infty ne^{9n}$$

is convergent or divergent.

19. (from 7.8 WebAssign, also in Week 7 sample quiz)
    (a) Find the values of p for which the integral $\displaystyle\int_e^{\infty} \frac{6}{x(\ln x)^p}$ dx converges. Evaluate the integral for these values of p.
    (Hint: u-substitution. Check what happens when $p = 1$, when $p < 1$, and when $p > 1$.)
    (b) Determine whether
    $$\int_2^{\infty} \left(\frac{1}{e^5}\right)^x \text{ dx}$$
    is convergent or divergent. If it is convergent, evaluate it.
    (c) Determine whether
    $$\int_2^{\infty} \frac{1}{x^2 + 8x - 9} \text{ dx}$$
    is convergent or divergent. If it is convergent, evaluate it.
    (d) Determine whether
    $$\int_0^{1} \frac{4}{x^5} \text{ dx}$$
    is convergent or divergent. If it is convergent, evaluate it.
    (e) Determine whether
    $$\int_0^{1} \frac{4}{x^{0.5}} \text{ dx}$$
    is convergent or divergent. If it is convergent, evaluate it.
    (f) Determine whether
    $$\int_2^{3} \frac{2}{\sqrt{3 - x}} \text{ dx}$$
    is convergent or divergent. If it is convergent, evaluate it.